STUDIES OF OPTICAL SYSTEMS CONTAINING LIQUID CRYSTALS AND
HOLOGRAPHIC OPTICAL ELEMENTS

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fulfillment of the requirements for the
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by

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CHAPTER 1

INTRODUCTION

In this dissertation both form birefringent holographic retarders and holographic reflectors are considered. The physical parameters of holographic retarders are varied and the resulting changes in retardation are presented. The holographic retarders are compared with the corresponding negative uniaxial retarders and good agreement is found. The physical parameters that are varied are dielectric profile, incident angle, thickness, and periodicity. It is found that the square wave dielectric profile has the largest retardation when compared with other dielectric profiles. By varying both the incident angle and thickness the holographic retarders are found to behave exactly as linear uniaxial retarder. As the periodicity of the holographic retarder is varied the wavelength dispersion of its birefringence varies, allowing the holographic retarder’s dispersion to be tuned by simply varying the periodicity. The wavelength dispersion of both slanted and unslanted holographic retarders is tuned to match a LC material. The tuned wavelength dispersion retarders are used to compensate both a VA LCD and a TN LCD. The dielectric profile of holographic reflectors is varied and the resulting spectra are compared with measured spectra. Holographic reflectors that have uniaxial layers are also considered. LCDs containing slanted holographic reflectors are optimized for the particular displays of interest.
One common problem noticed when viewing LCD’s off axis, either from the top, bottom, or any of the corners, is that the quality of the image seen decreases dramatically. This is a common complaint of those that own LCDs, either in a laptop, computer screen, or TV. This decrease in image quality off axis is referred to as the viewing angle problem. To improve the image quality off axis, retardation films are included in the LCD between the LC layer and the polarizer, Fig. 1-1. The typically used retarders are produced from stretched polymer films, where the optical properties of these films are controlled by the molecular structure of the material and it is therefore, difficult to engineer the exact retarder needed. It will be shown that the optical properties of form birefringent holograms can be engineered to match the LC’s very well.

In this dissertation the resulting properties of holographic retarders and reflectors are presented in chapters 2 through 6. While the tools need to calculate these results are presented in appendices A and B. Appendix A has a review of the Berreman method used to calculate light propagation along with the new extension of the Berreman method. This extension allows calculation of slanted holographic layers using the Berreman method. Appendix B reviews the program written to complete these calculations.
FIG. 1-1. In an uncompensated LCD, no retarders present, image quality decreases when the LCD is viewed off axis. The retarders on either side of the LC layer are used to cancel out the phase shift created upon propagation through the LC layer.
1.1 Photopolymer Holograms

Volume holograms are of interest to many fields for many different applications, such as; information storage, optical compensators, solar control reflective coatings, and are of particular interest to the liquid crystal display field as; form birefringent retarders, and rear reflectors on reflective displays. Holographic photopolymer films are being increasingly utilized for these applications. The technical basis of photopolymerization has been reviewed by Roffey \(^1\). Photopolymers “undergo physical, chemical or optical changes through selective polymerization when exposed to light.” \(^2\). The photopolymer film starts out as a homogeneous mixture of monomer, polymeric binder, and photoinitiator. Once the film is exposed to laser light, polymerization and monomer migration occurs. When photopolymers are illuminated by two coherent intersecting beams, constructive and destructive interference occurs, creating “bright” and “dark” fringes. In the “bright” fringes photoinitiators absorb light and the monomers begin to polymerize. As the polymer chains grow the monomer concentration decreases creating a density gradient, which prompts migration of the mobile unreacted monomers from the “dark” fringes to the “bright” fringes where they then polymerize \(^3\). The monomer and polymeric binder have different indices of refraction; therefore, creating an index modulation throughout the thickness of the film. After laser exposure is completed the photopolymer film is cured with UV “A” or white light to fix the hologram by consuming the unreacted material \(^4\). Once the UV cure has been finished the film is heated to further diffuse and polymerize the monomers, a step that amplifies the refractive index gradient and increases the reflection or transmission efficiency of the hologram.
In addition to this performance enhancement baking, an additional step can be added to tune the reflection or transmission peak to a desired wavelength. After the hologram is exposed and UV cured it is laminated to a “color-tuning film” (CTF) and heated. In this film are monomers that diffuse into the hologram and swell the grating layers changing the Bragg wavelength of the hologram, in other words the playback wavelength is red shifted. The playback spectrum can be controllably broadened by selecting the heating conditions ⁴.

As the hologram goes through each of these steps, the Bragg wavelength shifts because of a change in the index profile. For this reason, it would be useful to understand, through computer modeling, how changes in the index profile affect the hologram’s retardation, in the form birefringent regime, and the hologram’s spectra, in the Bragg regime. If the true shape of the index profile can be modeled, the optimum index profile for any given application can be found. All of the holograms considered in this dissertation are photopolymer, volume holograms.

There are many possible holographic recording materials. One of the most commonly known is photopolymer holographic materials, which are mentioned above. Some of the first materials used to write holograms are silver halide and dichromated gelatine. Both of these require wet chemical processing. In the case of silver halide holograms the bleaching process reduces the image contrast. Dichromated gelatine holograms shrink after processing and this could be used to create form birefringent holograms with a smaller periodicity then is possible with some of the other methods. Ferroelectric crystals such as Lithium niobate can also be used to record holograms.
Holographic layers are written on these materials using a combination of photosensitivity and electro-optic effect; therefore, new and interesting dielectric profiles could be created using this method. Photochromic and photodiechoic materials can also be used to write holographic layers. The material undergoes a reversible change between a colored state and a bleached state. Either state can be written on by exposing the film to light. There is a very interesting and thorough review of holographic recording materials by Solymar. All the holograms considered in this dissertation are photopolymer holograms.

1.2 Form Birefringent Retarders

Anisotropy can result in molecularly isotropic materials when there is a modulation of the refractive index, and the periodicity of that modulation is small compared to the illuminating wavelength of light. This phenomenon is well known and is referred to as form birefringence. In the form birefringent regime all higher order diffracted waves are evanescent; therefore, only the zeroth order diffraction exits the grating. The grating acts as a negative uniaxial retarder with the optic axis perpendicular to the index modulation.

The ideal compensator for an LCD would exactly match the optic axis orientation of the LC and the absolute value of the birefringence dispersion would equal the LC’s but of the opposite sign. Since form birefringent holographic retarders mimic negative birefringent retarders the second condition can be met. Matching the optic axis orientation for a VA LCD can be easily achieved using unslanted form birefringent holographic retarders. For the TN LCD, matching the optic axis orientation is not quite
as trivial when using holographic retarders. Recently, other polymeric retarders have emerged that have a continuously varying optic axis along the thickness of the retarder, such as the hybrid aligned LC polymerized retarder and the Fuji Wide View film. Another retarder considered is a simple stack of uniform optic axis retarders. These three types of retarders have the potential to mirror the optic axis orientation of the TN much better than previous retarders but the wavelength dispersion was not considered. The wavelength dispersion of quarterwave plates for LCD compensation is considered by many authors but these retarders do not mirror the TN’s optic axis and do not have negative birefringence. Form birefringent retarders allow for an arbitrary uniform optic axis orientation along with matched wavelength dispersion of the birefringence. Since holographic retarders can not have a splayed or twisted optic axis, several uniform optic axis retarders are stacked and the compensation results are compared with those of a splayed negative birefringent retarder.

The material properties of form birefringent retarders are constrained by the holographic materials used. Unlike the previously mentioned crystal and film retarders, there are many possible birefringence values and dispersion properties using just one holographic material. The birefringence, dispersion properties, and optic axis are varied by changing the hologram’s physical parameters instead of the molecular properties. The birefringence can be changed by varying the minimum and maximum indices of the dielectric profile or by changing the shape of the dielectric profile. The wavelength dispersion is varied by changing the periodicity of the dielectric profile. The
optic axis can be aligned in any desired orientation by varying the holographic layer’s slant angle.

1.3 Optical Modeling

Optical modeling is used prior to design completion in many fields for optimizing optical devices; therefore, simple, accurate, and fast computation methods are desired. Typical outputs of optical modeling are transmission, reflection, and polarization state. Such calculations are used to understand how device variations affect the spectra, luminance, and other light outputs that are specific to the device. For example, how the device output varies with voltage. In many cases, the optical modeling is based on mathematical methods such as the Berreman 4x4 method 17, Jones method 2x2 18, or even the use of Mueller matrices 19, which calculate how light interacts with the media of interest. All these methods assume that the material has a dielectric variation in only one dimension and that this can be discretized into layers in which the dielectric permittivity tensor is constant. Only the Berreman method can be used when considering coherent reflections. This restriction is somewhat limiting in several cases where the system otherwise fits the mathematical method well. Two examples of systems in which the dielectric variation occurs locally in only one dimension but globally in two dimensions are the modeling of birefringent beamsplitters in front of liquid crystal layers (for use in projection displays), and the modeling of liquid crystal layers in front of slanted holographic reflectors (see Fig. 1-2). The Berreman 4x4 method is used for the following calculations along with a new extension to the Berreman method that allows the
calculation to be extended into two dimensions. This extension can easily be incorporated into other calculation methods.
FIG. 1-2. The reference frame for each stack is defined such that the axis of dielectric variation is the x-axis.
1.4 Reflective Liquid Crystal Displays

There are two main categories for Reflective LCDs; those that function by switching polarization states and require polarizers and those that do not depend on switching polarization state and do not have polarizers. The latter, are either cholesteric displays or guest-host displays. Both were developed for high brightness due to the lack of polarizers. Reflective displays that depend on switching between states by varying the polarization of light can either have two polarizers, one at the front of the display and one immediately before the reflector, or only one polarizer at the front of the display. The single polarizer reflective display was devised to allow the reflector to be located inside the LC cell to eliminate parallax. When compared with the two polarizer reflective display the single polarizer display has the advantage of being brighter, due to the elimination of one polarizer. The possibility of including the reflector inside the LC cell is ideally suited to holographic reflective color filters, which combine the color filter and reflector in one film. Holographic slanted reflectors have the added advantage of reflecting light at an angle other then the incoming light’s incident angle, eliminating specular reflections that degrade the contrast ratio.

1.4.1 Low Twist Liquid Crystal Displays

Reflective LCDs can be constructed in several different configurations to create either a “Normally White” display or a “Normally Black” display. To decide which configuration is desired the color shift in both the bright and dark state are considered. A shift in color for the dark state is more noticeable because the addition of a single color,
for instance blue, is more easily perceived when added to black verses when a single color is added to white. Therefore, the dark state should be the state with lower birefringence, which is the high voltage state. The “Normally White” twist LCD is desirable because the state that has the lowest birefringence is the most achromatic. In a reflective LCD light travels through the LC layer twice increasing the total retardation, which increases the difference in phase shift between the colors reflected. For the reflective LCD to have similar achromatic properties to its transmissive counterpart it must be thinner. The total retardation, for both passes, of the reflective LC layer is approximately a half wave \(^{20}\). Thus, the single polarizer display with out compensators is “Normally Black.” The desired “Normally White” display is created by the addition of a quarter wave plate between the polarizer and LC layer in the single polaizer display; therefore, the polarization incident on the LC layer is circular. It is demonstrated that circular polarized light incident on lower twist displays allows for optimization of both the bright and dark state, while higher twist displays require consideration of the alternative input polarization states.

The low twist single polarizer display containing a quarter wave plate is investigated by generating parameter space diagrams similar to those used by Kwok \(^{19}\). The parameter space diagrams vary the LC’s twist angle, retardation, polarizer’s rotation angle, and the rotation of the entire polarizer, quarter wave plate, LC stack on top of the holographic slanted reflector. These contour plots have the reflectance as the height of the contours and allow for optimization of both the bright and dark state, which has not been considered by previous authors.
1.4.2 High Twist Liquid Crystal Displays

Liquid crystal displays with a twist angle greater than 180° can be driven as a passive matrix display, which has the attractive property of low power consumption. The propagation of monochromatic light along the LC’s helical axis is a superposition of two normal modes, which are elliptically polarized with the vibrational ellipse’s major axes parallel and perpendicular to the LC’s director \(^{21}\). When the twist of the LC layer satisfies the Mauguin condition the normal modes can be considered linearly polarized \(^{21}\). The Mauguin condition is:

\[
\Delta n d \gg \frac{\Phi \lambda}{\pi}
\]  

(1.1)

where \(\Phi\) is the LC layer’s total twist in radians, \(\lambda\) is the wavelength of light in vacuum, and \(d\) is the total LC layer’s thickness. This condition is only approximately met for twists less than 90°, which results in decreased brightness and increased chromaticity. For high twist displays the Mauguin condition is met even less. The optical design of low twist displays revolves around the approximation that the incident polarized light approximately follows the twist of the LC layer, in a waveguide effect \(^{21}\). For high twist displays this approximation begins to deteriorate and the situation is complicated by the addition of a mirror without a polarizer in front of it, which allows elliptically polarized light to reflect from the mirror. Upon reflection from a mirror there is a \(\pi\) phase shift, which changes the handedness of rotation of elliptically polarized light. This complicates optimizing the LC layer.
It has been shown that to create a high contrast high twist display with good achromaticity, optical compensators are needed\(^{22-24}\). However, the polarization optics are quite complex and defining the LC layer, its select and non-select voltages, along with the compensators retardation and angles creates a very large parameter space. To reduce the number of variables considered an optimization method is devised that simply follows the polarization states through the LCD. This optimization method allows the LC layer to be defined without knowledge of the compensators or polarizer. After the LC layer is optimized the rest of the display is optimized by finding the compensators and polarizer that generate the required input polarization state incident on the LC layer.
CHAPTER 2

VARYING THE STRUCTURE OF HOLOGRAPHIC RETARDERS

2.1 Modeling Holograms

The hologram modeling program simulates light propagation through a stack of layers that are uniform along the length of the layers. In the program the calculation begins with a cover layer of isotropic material and then enters the first layer of the hologram continuing through approximately 2000 layers of hologram media, which have a changing index of refraction, and finally ends in a rear layer of isotropic material. The hologram is discretized into many small layers of constant index therefore, the index of the hologram layers can modulate in any function or shape desired to simulate the actual index profile within the hologram. The number of layers within the hologram depends on how intricate and quickly changing the modeled index is within the hologram. Important parameters that must be input into the program are the total thickness of the hologram, the periodicity, the maximum and minimum indices of refraction, and the Bragg wavelength. Due to the Bragg condition four of these parameters are used to find the hologram's periodicity, $\Lambda$,

$$\Lambda = \frac{\lambda}{2 \cdot [(n_1 + n_2)/2] \cdot \sin \theta}$$

(2.1)

where $\lambda$ is the Bragg wavelength, $n_1$ and $n_2$ are the minimum and maximum refractive indices, and $\theta$ is the incident angle of the incoming light.
2.2 Dielectric Profiles

Holograms have several possible dielectric profiles. We will consider profile functions for two different types of holograms; those created using vapor deposition and those created with photopolymer film. Holograms created using vapor deposition have the simplest dielectric profile, a square wave, which is a stack of alternating layers. Photopolymer holograms have numerous possible dielectric profiles, we investigate the following profiles; sine wave, clipped sine wave, and chirped sine wave. All of these profiles are illustrated in chapter 4, Figs. 4-1 through 4-3. The typically assumed profile for photopolymer holograms is a sine wave.

One possible modification of the sine wave dielectric profile is a clipped sine wave dielectric profile. The clipped sine wave dielectric profile may occur during the heating process, when creating photopolymer holograms. As the hologram is heated the layers of high and low index may saturate due to numerous monomer migrations.

Another possible dielectric profile in photopolymer holograms is chirped. Chirped holograms have a variation in the periodicity of the hologram’s index profile as a function of the total hologram thickness. In the Bragg regime, this is a desirable hologram profile since it broadens the spectra and causes the hologram to reflect a larger range of wavelengths, which will be discussed in more detail in chapter 4. The chirped sine wave dielectric profile is investigated in the form birefringent regime to see if it is useful for retarders.

The chirped sine wave dielectric profile occurs when an extra step is added to the hologram creation process. The additional step is the lamination of a “Color Tuning
Film” (CTF), which contains monomers that diffuse into the hologram and swell the grating layers closest to the side of the hologram that has the CTF attached. An unchirped sine wave index profile is derived later in Eq. (2.7) and does not have the addition X term. A chirped hologram model assumes a clipped sine wave index profile, and the periodicity of the clipped sine wave varies exponentially throughout the total thickness of the hologram. The chirped sine wave index profile includes an extra term to vary the periodicity of the hologram.

\[ n = \left[ n_a + n_b \cdot \cos \left( \left( \vec{K} \cdot \vec{x} \right) \cdot X \right) \right] \]  

(2.2)

The extra term that varies the periodicity is:

\[ X = 1 - (1 - A) \cdot e^{-\frac{B \cdot m}{100}} \]  

(2.3)

where A represents the peak chirping value, B varies the uniformity of the chirp throughout the thickness of the hologram, and m is the hologram layer number that is being calculated (between 1 and 2000). For the graphs of form birefringent holograms with a chirped sine wave dielectric profile, the chirping variables are defined as; \( A = B = 0.6 \).

2.3 Calculating Birefringence of Holographic Retarders

2.3.1 Theoretical Birefringence Values Derived from Coupled Wave Theory

The retardation value of holograms in the form birefringence regime has been theoretically predicted for both the square wave dielectric profile and the sine wave dielectric profile. The ordinary permittivity is calculated by averaging the tangential component of the dielectric field \( \vec{D}_t \) over a single period of the hologram. The
extraordinary permittivity is calculated by averaging the normal component of the electric field ($\vec{E}_1$) over a single period of the hologram\textsuperscript{7}. The ordinary and extraordinary indices of the square wave dielectric profile are predicted to be\textsuperscript{26}.

\[ n_o = \sqrt{\frac{n_1^2 + n_2^2}{2}} \quad (2.4) \]

\[ n_{e\text{square}} = \sqrt{\frac{2n_1^2n_2^2}{n_1^2 + n_2^2}} \quad (2.5) \]

where $n_1$ is the index of one of the alternating layers and $n_2$ is the index of the other.

The most commonly assumed dielectric profile for photopolymer holograms is the sine wave. Not only is it straightforward to derive the coupled wave theory for the sine wave profile but this dielectric shape is also a likely periodic structure for photopolymer holograms. A sine wave dielectric profile has a sinusoidally modulated permittivity given by:

\[ \varepsilon(\tilde{x}) = \varepsilon_a + \varepsilon_b \cos(\tilde{K} \cdot \tilde{x}) \quad (2.6) \]

where $\varepsilon_a$ is the average dielectric permittivity (also called bias permittivity), $\varepsilon_b$ is the amplitude of the permittivity modulation, and $\tilde{K}$ is the grating vector as described in Kogelnik's coupled wave theory\textsuperscript{27}. The standard parameter definitions are illustrated in Appendix A, Fig. A-1. The index profile is found from the dielectric profile, by binomial expansion of the square root of Eq. (2.6).

\[ n = \sqrt{\varepsilon_a + \frac{\varepsilon_b}{2\sqrt{\varepsilon_a}}} \cos(\tilde{K} \cdot \tilde{x}) = n_a + n_b \cos(\tilde{K} \cdot \tilde{x}) \quad (2.7) \]

The average index is found from Eq. (2.7).
\[ n_a = \sqrt{\varepsilon_a} = \frac{n_1 + n_2}{2}, \quad (2.8) \]

where \( n_1 \) is the maximum index in the sinusoidal modulation and \( n_2 \) is the minimum. The amplitude of the index modulation is also found from Eq. (2.7).

\[ n_b = \frac{\varepsilon_b}{2\sqrt{\varepsilon_a}} = \frac{n_1 - n_2}{2} \quad (2.9) \]

Once again, the ordinary and extraordinary permittivities are calculated by averaging the field components over a single period of the hologram. Any periodic dielectric profile, including sine, results in Eq. (2.4) describing the theoretical value of the ordinary index. The extraordinary index is found, after calculating the average, by using the definition of the index modulation in Eq. (2.7) \(^\text{28}\).

\[ n_e^{\text{sin}} = n_o - \frac{n_b^2}{n_a} \quad (2.10) \]

The birefringence of the sine wave dielectric profile is predicted to be \(^7,\text{28}\):

\[ \Delta n^{\text{sin}} = n_e - n_o = \frac{n_b^2}{n_a}. \quad (2.11) \]

There are not theoretical predictions for the birefringence of holograms with the clipped or chirped sine wave dielectric profiles in the form birefringence regime. But the birefringence due to a hologram with these profiles can be calculated using the Berreman method.
2.3.2 Theoretical Birefringence Values Derived from Phase Retardation

The phase retardation, $\Gamma$, of an anisotropic medium varies with incident angle according to the following relation:

$$\Gamma = (k_{ez} - k_{oz})d,$$

(2.12)

where $k_{ez}$ is the z component of the wave vector for the extraordinary eigenmode, $k_{oz}$ is the z component of the wave vector for the ordinary eigenmode, and $d$ is the thickness of the retarder. The phase retardation is calculated using the Berreman method, which is described in more detail in A.1.2, along with a definition of the Berreman axes in Fig. A-2. In a uniaxial media, the refractive index of the ordinary mode, $n_o$, is constant and does not depend on the propagation direction. While the refractive index of the extraordinary mode, $n_{\text{eff}}(\theta_{\text{eff}})$, varies depending on the angle between the $k_{ez}$ wave vector and the optic axis. We will assume an ordinary index of refraction and calculate the extraordinary index of refraction to find the birefringence of the hologram. To define the effective extraordinary index, $n_{\text{eff}}(\theta_{\text{eff}})$, the optic axis orientation must be known. The optic axis of a form birefringent hologram is defined as parallel to the grating vector $\vec{K}$. When calculating the hologram’s birefringence, the hologram layers are unslanted and parallel to the surface of the hologram; therefore, the simulated optic axis is parallel to the surface normal. The effective extraordinary index of a uniaxial media with its optic axis parallel to the surface normal is:

$$n_{\text{eff}}^2 = \frac{n_e^2 n_o^2}{n_e^2 \cos^2(\theta_{\text{eff}}) + n_o^2 \sin^2(\theta_{\text{eff}})}$$

(2.13)
where $\theta_{\text{eff}}$ is the angle of incidence in the material corresponding to the effective extraordinary index and $n_e$ is the extraordinary index, which is a parameter of the material and does not change with incident angle. Both eigenmodes’ angles of incidence, $\theta_o$ and $\theta_{\text{eff}}$, obey Snell's law. Using the eigenmodes’ indices and angles, Eq. (2.12) is rewritten 6:

$$\Gamma = \frac{2\pi d}{\lambda} \left( n_{\text{eff}} \cos(\theta_{\text{eff}}) - n_o \cos(\theta_o) \right).$$  \hspace{1cm} (2.14)

Given that the phase retardation explicitly depends on wavelength, the retardation will be used instead to calculate the birefringence. The relation between the path length retardation, $R$, and the phase retardation is:

$$\Gamma = \frac{2\pi R}{\lambda}$$  \hspace{1cm} (2.15)

Eqs. (2.15) and (2.14) are used to solve for the effective extraordinary index. Inserting the value found for the effective extraordinary index into Eq. (2.13) the extraordinary index is calculated, resulting in a value for the birefringence:

$$\Delta n = (n_e - n_o).$$  \hspace{1cm} (2.16)

2.4 Comparing Form Birefringent Holographic Retarders to Negative Uniaxial Retarders

The phase retardation is calculated using the Berreman method by simulating light propagation through the hologram and any other components in the optical path. The incident light is polarized at 45°, where this is defined as 45° from the y-axis when the incident wave vector is parallel to the display’s normal and the plane of rotation is
always perpendicular to the wave vector. The axes used for the Berreman method are defined in Fig. 2-1. Within the calculation, the phase retardation is defined as the phase difference between the exiting orthogonal electric field components, $E_p$ and $E_s$. Light is incident at 60°, unless the parameter varied is incident angle. The wavelength used is 550 nm, unless the varied parameter is wavelength. The wavelength is defined in air and the surrounding isotropic media is chosen to reduce specular reflections. The incident and exiting electric fields are defined in the surrounding isotropic media, which is on both sides of the material of interest.

Two form birefringent holographic systems are investigated using the modeling software. The first hologram considered is the same as that studied by Eblen and associates. This system is examined to compare existing data with our calculated data. The hologram consists of alternating layers of titanium dioxide, TiO$_2$, ($n_1 = 2.13$) and silicon dioxide, SiO$_2$, ($n_2 = 1.53$). The layer thickness is 20 nm (40 nm periodicity) and the total thickness of the hologram is 1.11 μm (Fig. 10 in Ref. 26). The layers are unslanted and parallel to the surface of the hologram. The surrounding isotropic media has an index of 1.75 (for index matching to avoid specular reflections).

Both the theory for the square wave dielectric profile and for the sine wave dielectric profile assume that the hologram is well in the form birefringence regime such that, the retardation has reached a steady state solution and is fairly constant, Fig. 2-2. The plot in Fig. 2-2 shows the variation in the retardation ($R = \Delta n d$) as a function of wavelength divided by the periodicity of the hologram. The steady state solution for the retardation occurs after the average wavelength of light in the material is greater then
approximately 10 times the periodicity of the sample. There is a 6% variation between
the retardation at 10 times the periodicity, $\Delta nd_{10A} = -30.5\, nm$, and 24 times the
periodicity, $\Delta nd_{24A} = -28.4\, nm$. 

FIG. 2-1. Berreman axes definition. Each optical component’s optic axis is defined by its polar angle, $\psi$, and its azimuthal angle, $\xi$. The incoming light ray is described by a different polar angle, $\theta$, and azimuthal angle, $\varphi$. 
FIG. 2-2. Retardation versus wavelength divided by the periodicity of the hologram, where there is a 40 nm periodicity. The hologram is mentioned above and has a square wave dielectric profile, where, $n_1 = 2.13$ and $n_2 = 1.53$. The total thickness of the hologram is 1.11 μm. The wavelength is defined in air and the incident material has an index of $n = 1.75$. 

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Using the retardation from Fig. 2-2 the birefringence is calculated. We will assume an ordinary index of refraction of $n_o = 1.85$ and calculate the extraordinary index of refraction from the previous relations to find the birefringence of the hologram. The steady state retardation found in Fig. 2-2 is ~28.4 nm, using this value along with Eqs. (2.13) through (2.16) the birefringence is calculated to be $\Delta n^\text{square}_{\text{hologram}} = -0.1$, which agrees well with the predicted theoretical value found from Eqs. (2.4) and (2.5), $\Delta n^\text{square}_{\text{calc}} = -0.097$. Fig. 2-3 compares the phase retardation of a form birefringent hologram with a negative birefringent retarder, which has the calculated birefringence.

The second hologram investigated models a DuPont photopolymer hologram, HRF-700 28, and has n1 = 1.595, n2 = 1.449, d = 45.2 $\mu$m, a sine wave dielectric profile (unless otherwise stated) and unslanted layers which are parallel to the surface. The surrounding isotropic media has an index of 1.5. The plot in Fig. 2-4 shows the variation in retardation as a function of wavelength divided by the periodicity of the hologram. Three plots for different periodicities are shown in Fig. 2-4. Notice that once the wavelength is normalized to the periodicity all plots coincide. We will assume an ordinary index of refraction, $n_o = 1.5237$, and calculate the extraordinary index of refraction. The steady state retardation found in Fig. 2-4 is ~63 nm, using this value along with Eqs. (2.13) through (2.16) the birefringence is calculated as $\Delta n^\text{sin}_{\text{hologram}} = -0.0035$, which agrees well with the predicted value found from Eq. (2.11), $\Delta n^\text{sin}_{\text{calc}} = -0.0035$. Fig. 2-5 compares the phase retardation of a form birefringent hologram with a negative birefringent retarder, which has the calculated birefringence. Notice that there is very
good agreement in phase shift between the form birefringent hologram and the uniaxial retarder.
FIG. 2-3. Phase shift versus incident angle in air for both a uniaxial retarder and a form birefringent hologram with a square wave dielectric profile. The parameters of the uniaxial retarder are $n_o = 1.85$, $n_e = 1.75$, $d = 1.11 \mu m$, the optic axis is parallel to the film normal. The hologram is mentioned above and has a square wave dielectric profile, where, $n_1 = 2.13$ and $n_2 = 1.53$. A total thickness of 1.11 $\mu m$ and a 40 nm periodicity.
FIG. 2-4. Retardation versus wavelength divided by the periodicity of each hologram.

Where there are 3 holograms with periodicities equal to 40 nm, 100 nm, and 150 nm. Holograms have a sine wave dielectric profile. Light is incident at 60°. The wavelength is defined in air and the incident material has an index of $n = 1.5$. 
FIG. 2-5. Phase shift versus incident angle in air for both a uniaxial retarder and a form birefringent hologram with a sine wave dielectric profile. The parameters of the uniaxial retarder are $n_o = 1.5237$, $n_e = 1.5202$, $d = 45.2 \mu m$, the optic axis is parallel to the film normal. This hologram models the DuPont hologram mentioned above.
2.5 Effect of Varying Hologram Dielectric Profile on Form Birefringence

The retardation of a form birefringent hologram changes depending on the dielectric profile in the hologram. In Fig. 2-6, four different form birefringent holograms are compared. All holograms have the same parameters, \(n_1 = 1.595\), \(n_2 = 1.449\), \(d = 45.2\) \(\mu\)m, unslanted layers, and incident index equal to 1.5; except the dielectric profiles have been changed. The various dielectric profiles are sine wave, clipped sine wave, chirped sine wave, and square wave. The more distinctly defined the hologram layers are, the higher the birefringence. Notice that the square wave dielectric profile has the largest retardation and the sine wave has the smallest. The birefringence of a form birefringent hologram can be increased by creating more distinctly defined holographic layers. Using Eq. (2.16), the birefringence of the square wave dielectric profile hologram is predicted to be \(\Delta n_{\text{Square}^2_{\text{Bireman}}} = -0.0074\), \((\Delta n_{\text{Square}^2_{\text{Calc}}} = -0.007)\) which is almost twice the value predicted for the sine wave dielectric profile using Eq. (2.16), \(\Delta n_{\text{Sin}_{\text{Bireman}}} = -0.0035\).
FIG. 2-6. Retardation versus wavelength divided by periodicity for various functional forms of the dielectric profile in the hologram. The wavelength is defined in air and the incident material has an index of $n = 1.5$. 
2.6 Effect of Varying Incident Angle on Form Birefringence

The angular dependence of the phase shift is considered. The following graphs are for the hologram that models the DuPont photopolymer hologram, the periodicity and the dielectric profile are varied. Fig. 2-7 plots the angular dependence of the phase shift for holograms with varying periodicities. The phase shift is larger for the hologram with the greatest periodicity, as is expected for the hologram with the largest overall birefringence. Fig. 2-8 plots the angular dependence of the phase shift for several dielectric profiles. The square wave dielectric profile has the largest phase shift, as is expected for the hologram with the largest overall birefringence.
FIG. 2-7. Phase shift versus incident angle for various hologram periodicities. All three holograms have a sine wave dielectric profile.
FIG. 2-8. Phase shift versus incident angle for various functional forms of the dielectric periodicity in the hologram. All four holograms have 50 nm periodicity.
2.7 Effect of Varying Hologram Thickness on Form Birefringence

Varying the total hologram thickness changes the phase shift in a linear manner. Linear retarders exhibit this type of linear change in phase shift. The following graphs are for the hologram that models the DuPont photopolymer hologram, the periodicity and the dielectric profile are varied. Fig. 2-9 plots the change in hologram thickness versus the phase shift for holograms with varying periodicities. Once again, the phase shift is larger for the hologram with the greatest periodicity. Fig. 2-10 plots the change in hologram thickness versus the phase shift for several dielectric profiles. Once again, the square wave dielectric profile has the largest phase shift. The clipped sine wave and the chirped sine wave dielectric profile holograms have the same phase shift until they are about 42 μm thick, then the chirped sine wave profile hologram has a lower phase shift then the clipped sine wave profile hologram. This is due to the fact that the chirping function that creates the hologram chirps a percentage of the total hologram thickness.
FIG. 2-9. Phase shift versus total hologram thickness for various hologram periodicities.
FIG. 2-10. Phase shift versus total hologram thickness for various functional forms of the dielectric periodicity in the hologram. Wavelength is 550 nm and the periodicity is 100 nm.
2.8 Effect of Varying Hologram Periodicity on Form Birefringence

The wavelength dispersion of birefringence for form birefringent holograms can be tuned by varying the hologram’s periodicity with respect to the wavelength range of interest. As the hologram’s periodicity increases the range of retardation values increases also. This continues until the hologram is in the Bragg regime. In chapter 3, the hologram’s wavelength dispersion is tuned to match that of a LC layer and the results are presented. The following graphs are for the DuPont photopolymer hologram, the periodicity and the dielectric profile are varied. Fig. 2-11 continuously varies the hologram’s periodicity until the Bragg regime is reached. The largest periodicity possible, for the conditions stated, is 200 nm for all the dielectric periodicities considered. Fig. 2-12 shows the reflectance of the holograms considered in Fig. 2-11.

Fig. 2-13 illustrates phase shift versus wavelength for a constant retardation of -63 nm, this illustrates how the implied wavelength dependence inherent in phase shift effects the plot, as shown in Eq. (2.15). A sample of the possible dispersions is shown in Figs. 2-14 and 2-16. Fig. 2-14 demonstrates that as the hologram’s periodicity increases the curvature and change in phase shift from the hologram also increases. This results in various possible wavelength dispersions of birefringence, which can then be matched to the LC that is being compensated. Fig. 2-16 reveals that the wavelength dispersion of birefringence is unaltered as the dielectric profile is changed. Therefore, the same wavelength dispersion could be attained using a compensator with a square wave dielectric profile as with a sine wave but the square wave has the added advantage of a higher average birefringence. Figs. 2-15 and 2-17 show the reflectance for the same
holograms used in Figs. 2-14 and 2-16. The visible spectrum is well beyond the Bragg regime for each of the holograms considered.
FIG. 2-11. Phase shift versus hologram periodicity for various functional forms of the dielectric periodicity in the hologram.
FIG. 2-12. Reflectance versus hologram periodicity for various functional forms of the dielectric periodicity in the hologram.
FIG. 2-13. Phase Shift versus wavelength for a constant retardation of -63 nm. This plot demonstrates the implied wavelength dependence inherent in phase shift, as shown in Eq. (2.15).
FIG. 2-14. Phase shift versus wavelength for various hologram periodicities. All holograms have the sine wave dielectric profile. 60° incident. The wavelength is defined in air and the incident material has an index of $n = 1.5$. 

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FIG. 2-15. Reflectance versus wavelength for various hologram periodicities. 60° incident angle. The wavelength is defined in air and the incident material has an index of $n = 1.5$. 
FIG. 2-16. Phase shift versus wavelength for various functional forms of the dielectric periodicity in the hologram. All holograms have a 50 nm periodicity. 60° incident angle.

The wavelength is defined in air and the incident material has an index of n = 1.5.
FIG. 2-17. Reflectance versus wavelength for various functional forms of the dielectric periodicity in the hologram. 60° incident angle. The wavelength is defined in air and the incident material has an index of $n = 1.5$. 
CHAPTER 3

APPLICATIONS OF HOLOGRAPHIC RETARDERS

3.1 Compensating a Vertically Aligned LCD

To demonstrate that a form birefringent hologram can have its dispersion tuned to match the dispersion of the LC material, and that the matched dispersion retarder compensates the LC more effectively, we will consider a vertically aligned (VA) LCD. A VA LCD has the LC molecules oriented so that their average effective optic axis is parallel to the layer normal. A VA LCD is ideally compensated by a negative birefringent retarder with its optic axis parallel to the layer normal. Form birefringent holograms with unslanted layers that are parallel to the hologram’s surface have an effective optic axis parallel to the layer normal. The display considered consists of crossed polarizers, a LC in the VA mode, and a compensator. This is a normally black configuration; therefore, when the display is off it appears dark. The form birefringent hologram compensates the dark state. Compensating the dark state with a dispersion matched compensator reduces the color shift, which is much more apparent in the dark state then in the bright state.

The LC material modeled is MLC-6608, its cauchy coefficients are \( \varepsilon_0 = 1.4643 \), \( B_0 = 3995 \text{ nm}^2 \), \( \varepsilon_1 = 1.5370 \), \( B_1 = 7509 \text{ nm}^2 \), the dielectric constants are; \( \varepsilon_{||} = 3.6 \), \( \varepsilon_{\perp} = 7.8 \), and the elastic constants are; \( k_{11} = 16.7 \text{ pN} \), \( k_{33} = 18.1 \text{ pN} \). The LC has a 90° pretilt and the cell is 3.26 \( \mu \text{m} \) thick. The form birefringent hologram used to compensate the
LC models a DuPont photopolymer hologram, HRF-700 \(^{28}\), and has \(n_1 = 1.595\), \(n_2 = 1.449\), \(d = 69 \, \mu\text{m}\), a sine wave dielectric profile (unless otherwise stated), the hologram’s periodicity is 66 nm, and unslanted layers which are parallel to the surface. The surrounding isotropic media has an index of 1.5. The only deviations from the previous description (chapter 2) are that the thickness is increased so that the retardation of the hologram equals that of the LC layer, and the hologram’s periodicity is defined by matching the hologram’s wavelength dispersion to the LC’s. The hologram’s wavelength dispersion is tuned by varying its periodicity. The first variable defined when designing a hologram to compensate the LC is the total thickness. The thickness is found by matching the hologram’s retardation to the LC’s. The indices are predetermined by the material parameters, \(n_1\) and \(n_2\). After the thickness and indices are defined the hologram’s periodicity is varied until the resulting path length retardation of the hologram matches that of the LC for all wavelengths. The form birefringent hologram is compared with a standard negative birefringent retarder made of discotic LCs. The optic axis of the negative birefringent retarder is parallel to the film normal (negative c-plate); its estimated cauchy coefficients are \(A_0 = 1.5441\), \(B_0 = 5163.4 \, \text{nm}^2\), \(A_e = 1.5\), \(B_e = 0 \, \text{nm}^2\), and the thickness is 4.47 \(\mu\text{m}\).

The dispersion of the form birefringent hologram is shown to match that of the VA LC in Fig. 3-1, while the dispersion of the negative c-plate has a greater variation with wavelength. Figs. 3-2 and 3-3 show the transmission through the LCD compensated with either the form birefringent hologram or the negative c-plate. At 30° off axis, the transmission is slightly decreased when the form birefringent hologram is used. At 60°
off axis, the transmission is noticeably decreased with the form birefringent hologram, due to total compensation at all wavelengths by this compensator. The transmission through crossed polarizers equals the transmission through the LCD compensated with the dispersion matched hologram, at 60° off axis. There is almost perfect compensation at 60° because the retardation matching was optimized at this angle. At 30° off axis the transmission through crossed polarizers is less then the transmission through the LCD compensated with the hologram because the retardation dispersion is not exactly matched at this angle of incidence. The transmission could be minimized for all angles and all wavelengths if the retardation of the retarder is the same as the LC at all angles. For this to occur the thickness of the hologram must equal the LC’s and the ordinary index of the hologram must equal the extraordinary index of the LC and vice versa. The system we have demonstrated has a slight increase in transmission when compared with the crossed polarizer transmission, since we are modeling a form birefringent hologram that can be created in the lab and not an ideal form birefringent hologram. However, the dispersion matched form birefringent retarder has better results than the negative c-plate.
FIG. 3-1. Retardation versus wavelength. The incident angle is 60°. The retardation shown for both the form birefringent hologram and the negative c-plate are the absolute value of the retardation, the actual retardations for these two materials are negative. The absolute value of the retardation for the form birefringent hologram equals the retardation of the LC. The dispersion of the negative c-plate has a greater change with wavelength then the dispersion of the LC.
FIG. 3-2. 30° incident angle. The VA compensated with the dispersion matched form birefringent hologram is better compensated than the VA compensated with the negative c-plate. The maximum transmission for the above graph is 100.
FIG. 3- 3. 60° incident angle. The VA compensated with the dispersion matched form birefringent hologram has the same transmission as through crossed polarizers. The VA compensated with the negative c-plate has a higher transmission. The maximum transmission for the above graph is 100.
3.2 Compensating a Twisted Nematic LCD

A TN-LCD is compensated using a stack of negative birefringent retarders that can have any uniform optic axis orientation through out the thickness of the retarder. Four different configurations are investigated; a single layer retarder, a stack of two retarders, a stack of three retarders, and a stack of four retarders. All configurations have the same stack on either side of the LC. The TN-LCD is first compensated by ideal retarders to examine the results from a configuration that has retarders with the same birefringence and dispersion as the LC material. Those results are then compared with an LCD compensated by a stack of form birefrigent holographic retarders.

3.2.1 Matching the Retarder's Optic Axis to the LC's

To ideally compensate a LC the retarder's ordinary refractive index, \( n'_o \), must equal the LCs extraordinary, \( n'_e \), and the retarder's extraordinary index, \( n'_e \), must equal the LCs ordinary, \( n'_o \). \(^{31, 32}\)

\[
\begin{align*}
    n'_o &= n'_e = n_o \\
    n'_e &= n'_o = n_b
\end{align*}
\]  

(3.1)

There is perfect compensation when the optic axes of the retarder exactly mirror the optic axes of the LC and the extraordinary propagation mode sees the extraordinary index, \( n_e \), and not the effective extraordinary index, \( n_{ef} \). Perfect compensation does not occur when the extraordinary propagation mode sees the effective extraordinary index because \( n_{ef} \) is different for the retarder and LC.
\[ n_{\text{eff}}^f = \frac{n_a n_t}{\sqrt{n_a^2 \cos^2(\eta) + n_t^2 \sin^2(\eta)}} \]  

(3.2)

and

\[ n_{\text{eff}}^c = \frac{n_a n_b}{\sqrt{n_a^2 \cos^2(\chi) + n_b^2 \sin^2(\chi)}} \]  

(3.3)

where \( \eta \) is the angle between the wave vector and the retarder’s optic axis and \( \chi \) is the angle between the wave vector and the LC’s optic axis. After light has propagated through the LC and retarder there is a phase shift, \( \Gamma \). Complete compensation occurs when that phase shift equals zero \(^{31, 32}\). The phase shift for light propagating perpendicular to the surface of the LC and retarder is:

\[ \Gamma = \frac{2\pi d}{\lambda} \left[ n_{\text{eff}}^c - n_b + n_{\text{eff}}^f - n_a \right]. \]  

(3.4)

The retarder’s ideal optic axis polar angle, \( \eta \), is found by setting eq. (3.4) equal to zero. The polar angles for the ideal retarders and the form birefringent retarders are found using this method. The azimuthal angles of the ideal retarders and form birefringent retarders are equal to the averaged azimuthal angle of the LC.

3.2.2 Ideal Layered Retarders

Four different compensating stacks are investigated, where those stacks consist of layering 1, 2, 3, or 4 negative birefringent retarders on both sides of the LC. The layered negative birefringent retarders are ideal, meaning that the Cauchy coefficients are the same as the LC material’s except that the ordinary and extraordinary variables are exchanged; \( A_o = 1.5367, B_o = 7991 \text{ nm}^2, A_e = 1.4588, \) and \( B_e = 4985 \text{ nm}^2 \). The
retarders compensate the dark state of the TN, which is at 5 volts. The retarders’ optic axes are uniform and the polar and azimuthal angles are defined by grouping the TN’s 41 layers into twice the number of layers in the retarder stack. The polar and azimuthal angles in each group of layers are averaged, and those values are then used in eq. (3.4), resulting in the retarders’ optic axes angles. The optic axis orientation and retarder thickness vary according to the number of retarders in the stack, see Table I. The stacked retarders have each layer of retarder separated; therefore, not only the polar angle can be adjusted to match that of the LC’s but the azimuthal angle of each separate retarder’s optic axis can also be adjusted.

The LC material modeled is MLC-7700-000, its Cauchy coefficients are: \(A_0 = 1.4588\), \(B_0 = 4985 \text{ nm}^2\), \(A_e = 1.5367\), \(B_e = 7991 \text{ nm}^2\), the dielectric constants used are; \(\varepsilon_\parallel = 4\), \(\varepsilon_\perp = 12\), and the elastic constants are; \(k_{11} = 13.2 \text{ pN}\), \(k_{22} = 6.4\), and \(k_{33} = 19.5 \text{ pN}\). The pretilt is 3° and the cell is 5.42 \(\mu\text{m}\) thick.
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<th>Azimuthal angle, $\phi$ (°)</th>
<th>Thickness, $d$ (µm)</th>
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<td></td>
<td>53.79</td>
<td>0.09</td>
<td>0.661</td>
</tr>
</tbody>
</table>

Table I. Optic axis orientation and thickness of layered negative birefringent retarders.

The ideal negative birefringent retarder stacks are compared with a standard negative birefringent retarder made of discotic LCs. The optic axis of the negative birefringent retarder is splayed linearly along the film normal with a single azimuthal orientation; its estimated Cauchy coefficients are: $A_0 = 1.5441$, $B_0 = 5163.4$ nm$^2$, $A_e = 1.5$, $B_e = 0$ nm$^2$, and is 1.4 µm thick unless otherwise stated.$^{10}$ A negative C-plate (TAC) is on one side of the splayed retarder, the side with the lowest polar angle, its dispersion is found in Ref.$^{33}$, and is 230 µm thick.$^{10}$ Unlike the stacked retarders, the
splayed retarder has only one azimuthal angle, which is aligned with the TN’s rub direction on both the top and bottom of the cell.

The optical stack consists of a polarizer (azimuthal angle 90°), retarder stack, LC layer (azimuthal angle of top rub at 0°), retarder stack rotated 90°, and an analyzer (azimuthal angle 0°), Fig. 3-4. The polarizers are arranged so that the transmission axis is perpendicular to the extraordinary axis of the retarders and the rub direction of the LC cell; therefore, light propagates along the O-mode of the retarders and LC. For the transmission versus wavelength graphs, the incident polar angle is either 0°, 30°, or 60° in air and the incident azimuthal angle is 0°. For the isocontrast plots the wavelength is 550 nm in air.

Figs. 3-5 through 3-10 show the transmission versus wavelength for the display’s dark state, the TN at 5 volts. Each graph shows the results for one angle of incidence, 0°, 30°, 60°. In each graph, the transmission through the display compensated by the pair of four proposed retarder stacks are compared with the transmission through the display compensated by the pair of splayed discotic retarders, the uncompensated TN LCD, and crossed polarizers. The transmission of both the dark and bright states are normalized so that the transmission through parallel polarizers is 50%. For the three incident angles of propagation in the material, the display compensated by the pair of 4 layer negative birefringent retarders has the least light leakage of all compensated displays. On axis, the display compensated by the pair of single layer negative birefringent retarders actually increases the light leakage when compared with the uncompensated TN. For all three incident angles, both the display compensated by the pair of 3 layer and 4 layer retarder
stacks show less light leakage then the display compensated by the pair of splayed retarders.

Figs. 3-11 through 3-13 present the transmission versus wavelength for the display’s bright state, the TN at 0 volts. Each graph shows the results for one angle of incidence, 0°, 30°, or 60°. Similar to the graphs for the dark state, all graphs compare the transmission through several different displays; those compensated by the 4 proposed retarder stacks, those compensated by a splayed discotic retarder, an uncompensated TN-LCD, and parallel polarizers. On axis, all display configurations result in similar transmission versus wavelength results. When the angle of incidence is 30° and 60°, the uncompensated display is the brightest. This is an expected result since all the retarders discussed have been created to compensate the dark state and will therefore, result in some loss to the state not compensated. When comparing the compensated displays, the splayed retarder display shows the highest brightness and the 4 retarder stack display has the next best results for the bright state. The display compensated by the splayed retarder is brighter because the total retardation of that retarder is slightly less then the retardation of the stacks of retarders proposed, which were created to have the exact same total retardation as the LC layer. It has been found that a retarder with slightly lower retardation has an increase in brightness of the bright state with a very small increase in light leakage for the dark state.

The isocontrast plots for the configurations mentioned earlier are shown in Figs. 3-14 through 3-19, the wavelength used for calculation is 550 nm. The viewing cone of the uncompensated TN is expanded in the horizontal direction by the 1 layer retarder stack,
on either side of the LC, but is decreased in the vertical direction; therefore, this retarder stack is not a viable solution for compensating TN-LCDs. The 2 layer, 3 layer, and 4 layer retarder stacks increase the viewing cone of the display when compared with the uncompensated TN. The display compensated by the pair of 2 layer retarders has a more uniform viewing cone than that of the display compensated by the pair of splayed retarders, although the viewing cone of the display with the 2 layer retarder is slightly smaller. Both displays compensated by the 3 layer retarder stack and by the 4 layer retarder stack not only have a more uniform viewing cone than the display compensated by the splayed retarder but have a larger viewing cone also. All the stacked retarders investigated have the optic axis angles defined by the LC's optic axis using eq. (3.4); therefore, stacks that may not perform well here, such as the 2 layer retarder, may still have a much better solution that could be found by varying the optic axis angles and retarder thicknesses \(2^1\).
FIG. 3-4. Optical stack orientation.
FIG. 3-5. TN at 5 Volts, compensated by the various retarder stacks. Incident polar angle 0°.
FIG. 3-6. TN at 5 volts, enlargement of Fig. 3-5. Incident polar angle 0°.
FIG. 3-7. TN at 5 Volts, compensated by the various retarder stacks. Incident polar angle 30°.
FIG. 3-8. TN at 5 volts, enlargement of Fig. 3-7. Incident polar angle 30°.
FIG. 3-9. TN at 5 Volts, compensated by the various retarder stacks. Incident polar angle 60°.
FIG. 3-10. TN at 5 volts, enlargement of Fig. 3-9. Incident polar angle 60°.
FIG. 3-11. TN at 0 Volts. Incident polar angle 0°.
FIG. 3-12. TN at 0 Volts. Incident polar angle 30°.
FIG. 3-13. TN at 0 Volts. Incident polar angle 60°.
FIG. 3-14. Uncompensated TN. The contrast ratio values vary from 10 to 200 to 1 contrast and are identified by the legend above.
FIG. 3-15. 1 layer of ideal negative birefringence retarders on both sides of the LC layer.

The contrast ratio values vary from 10 to 200 to 1 contrast and are identified by the legend above.
FIG. 3-16. 2 layers of ideal negative birefringence retarders on both sides of the LC layer. The contrast ratio values vary from 10 to 200 to 1 contrast and are identified by the legend above.
FIG. 3-17. 3 layers of ideal negative birefringence retarders on both sides of the LC layer. The contrast ratio values vary from 10 to 200 to 1 contrast and are identified by the legend above.
FIG. 3-18. 4 layers of ideal negative birefringence retarders on both sides of the LC layer. The contrast ratio values vary from 10 to 200 to 1 contrast and are identified by the legend above.
FIG. 3-19. 1 layer of splayed negative birefringence retarder and 1 layer of TAC on both sides of the LC layer. The contrast ratio values vary from 10 to 200 to 1 contrast and are identified by the legend above.
3.2.3 Form Birefringent Layered Retarders

The results from compensating the TN with layered form birefringent holographic retarders are presented below. In this case, form birefringent retarders are constrained by the maximum and minimum refractive indices in the hologram. The hologram investigated models a DuPont photopolymer hologram, HRF-700\textsuperscript{28}, and has $n_1 = 1.595$, $n_2 = 1.449$, $\Lambda = 56.6$ nm, a sine wave dielectric profile, and the thickness varies. A complete description of the dielectric profile and how the slanted hologram is calculated is given in chapter 2 and appendix A. The hologram's wavelength dispersion is tuned to match that of the LC by varying the hologram's periodicity.

Fig. 3-20 shows the retardation of the LC material, hologram, and discotic retarder at an incident angle of 60°. To match the hologram's dispersion to the LC's the optic axes of each are perpendicular to the surface and the hologram's total retardation is set equal to the LC's, resulting in the hologram being 124 $\mu$m thick, for this calculation. The form birefringent hologram is compared with a standard negative birefringent retarder made of discotic LCs, as mentioned earlier. To compare the discotic retarder's retardation with the LC's and hologram's, its optic axis is also perpendicular to the surface and the retardation at 550 nm is set equal to the LC's retardation, resulting in the discotic retarder being 7.74 $\mu$m thick, for this calculation. Notice that the hologram's retardation is tuned to exactly match the LC's at an incident angle of 60°.

Figs. 3-21 through 3-23 present the transmission versus wavelength for the display's dark state, the TN at 5 volts. Each graph shows the results for one angle of incidence, 0°, 30°, or 60°. In these graphs a display compensated by a retarder stack
consisting of 3 form birefringent holographic retarders is shown in comparison with displays compensated by the splayed retarder and the 3 layer ideal retarder. The optic axis angles of the holographic retarders are the same as the ideal retarders, Table I. For all incident angles the display compensated by the 3 layer hologram has the least light leakage in the dark state.

Figs. 3-24 through 3-26 present the transmission versus wavelength for the display’s bright state, the TN at 0 volts. Each graph shows the results for one angle of incidence, 0°, 30°, or 60°. As in the graphs above, a display compensated by a retarder stack consisting of 3 form birefringent holographic retarders is shown in comparison with displays compensated by the splayed retarder and the 3 layer ideal retarder, all retarder stacks have identical stacks on both sides of the LC. On axis, every display shows similar results with out much variation created by the compensation stack. At incident angles of 30° and 60° the display compensated by the splayed retarder is the brightest, while the display compensated by the 3 layer hologram is the next brightest. The display compensated by the 3 layer hologram has a brighter bright state then the display compensated by the 3 layer ideal retarders because the hologram has been tuned to have the same birefringence as the LC but not the same indices; therefore, the path lengths in the holograms are different then that in the LC.

Fig. 3-27 shows the isocontrast plot for the display compensated by the 3 layer hologram stack. The display compensated by the 3 layer hologram stack has the largest viewing cone of all displays investigated. However, the viewing cone of the display compensated by the 3 layer hologram stack is not as uniform as the displays compensated
by the 2, 3, or 4 layer ideal retarder stack. This is due to the path length inside the holograms being different then inside the LC and because the hologram’s birefringence dispersion is matched to that of the LC at only an incident angle of 60°. At other angles of propagation in the hologram, the hologram’s birefringence dispersion closely matches the LC’s but is not exact\textsuperscript{16}.
FIG. 3-20. Retardation dispersion with respect to wavelength for the LC material used, the hologram, and the material used for the splayed retarder. Notice that the hologram is tuned to have the same dispersion as the LC. For this calculation, all optic axes are uniform and normal to the display surface. The hologram is 124 μm thick. The retarder is 7.74 μm thick.
FIG. 3-21. TN at 5 Volts. Incident polar angle 0°.
FIG. 3-22. TN at 5 Volts. Incident polar angle 30°.
FIG. 3-23. TN at 5 Volts. Incident polar angle 60°.
FIG. 3-24. TN at 0 Volts. Incident polar angle 0°.
FIG. 3-25. TN at 0 Volts. Incident polar angle 30°.
FIG. 3-26. TN at 0 Volts. Incident polar angle 60°.
FIG. 3-27. 3 layers of form birefringent holograms on both sides of the LC layer. The contrast ratio values vary from 10 to 200 to 1 contrast and are identified by the legend above.
CHAPTER 4

VARYING THE STRUCTURE OF HOLOGRAPHIC REFLECTERS

4.1 Dielectric Profiles

4.1.1 Square, Sine, and Clipped Sine Wave Dielectric Profiles

The simulated data below is calculated by inputting the hologram’s indices, periodicity, total thickness, and layer slant angle along with entering the index profile of the desired function. All of the graphs presented in this chapter have the hologram slanted so that $h = 14^\circ$, the input light is at $\theta = 35^\circ$, $\theta'' = 21^\circ$ (for these calculations $\phi$ is held constant at $0^\circ$), and the output light is at $\theta^R = 7^\circ$ (Fig. A-4 (a) and (b)). The graphs have profiles that are either a square wave, sine wave, or a clipped sine wave. The square wave is a basic Bragg reflector, a hologram made from photopolymers will not have this profile. The assumed profile for photopolymer holograms is a sine wave.

As seen below, the calculated reflectance maximum for a square wave profile (88%, Fig. 4-1a) is greater than the calculated reflectance maximum for a sine wave profile (75%, Fig. 4-2a). The actual reflectance maximum is greater than the spectrum generated with the sine wave profile. The clipped sine wave has a higher calculated reflectance maximum (85%, Fig. 4-3a), which is closer to what is expected for photopolymer holograms.
To evaluate correspondence with the more traditional coupled wave theory, the sine wave index profile was calculated using that method \(^{27}\). Results are shown in Fig. 4-4. The coupled wave theory and Berreman method agree well.
FIG. 4-1. (a) Hologram spectra for square wave index profile and (b) square wave index profile. The grating periodicity is 0.2 µm, the total grating thickness is 10.4 µm, $(n_2-n_1) = 0.059$, $n_1 = 1.4783$, $n_2 = 1.5217$, $\lambda_b = 514$ nm, 2000 layers. The light source has a 2 nm linewidth.
FIG. 4-2. (a) Hologram spectra for sine wave index profile and (b) sine wave index profile. The grating periodicity is 0.2 μm, the total grating thickness is 10.4 μm, \((n_2-n_1) = 0.059\), \(n_1 = 1.4783\), \(n_2 = 1.5217\), \(\lambda_b = 514\) nm, 2000 layers. The light source has a 2 nm linewidth.
FIG. 4-3. (a) Hologram spectra for clipped sine wave index profile and (b) clipped sine wave index profile. The grating periodicity is 0.2 μm, the total grating thickness is 10.4 μm, (n_2 - n_1) = 0.059, n_1 = 1.4783, n_2 = 1.5217, λ_b = 514 nm, 2000 layers. The light source has a 2 nm linewidth.
FIG. 4-4. Hologram spectra for sine wave index profile calculated with the Kogelnik method and Berreman method. The grating periodicity is 0.2 μm, the total grating thickness is 10.4 μm, \((n_2-n_1) = 0.059\), \(n_1 = 1.4783\), \(n_2 = 1.5217\), \(\lambda_b = 514\) nm, 2000 layers.

The light source has a 2 nm linewidth.
4.1.2 Chirped Sine Wave

Chirped sine wave dielectric profile holograms, in the Bragg regime, are of interest because the span of periodicities increases the bandwidth of reflected light. Using the chirped sine wave dielectric profile equation from chapter 2, Eq. (2.3), the parameter A is varied and the resulting spectra are compared, Figs. 4-5 through 4-8. The variable B, from Eq. (2.3), is set equal to 0.4 since this gives a uniform exponential change in the periodicity through out the thickness of the hologram. Varying the parameter A changes the peak periodicity of the chirped hologram. The smaller the variable A the thicker the widest periodicity. The approximate values of A and B for an actual chirped hologram are optimized by comparing the results of Figs. 4-5 through 4-8.

To further understand how to best model a chirped hologram, the parameters A and B are further varied and the resulting spectra is tuned until it matches actual hologram spectra, Fig. 4-9. The optimized calculated spectra, Fig. 4-10, was generated with the parameters A = B = 0.6. The optical stack used in Figs. 4-9 and 4-10 consists of a polarizer, quarterwave plate, LC, and reflective hologram. Light is incident on the optical stack at 35°. The calculated data that is reported for each wavelength is averaged over 20 nm to simulate the white light source used in the lab. The polarizer absorbing axis is aligned at 10°, the quarterwave plate’s slow axis is aligned at 90° and the LC’s top rub is aligned at 270° to the hologram’s plane of incidence.

The polarizer is an optical grade glass polarizing filter made by Prinz. Specifications for this polarizer are unavailable, estimated parameters are: thickness = 3.3 μm, no = 1.51+i*1.6e-3, and ne = 1.53+i*1.23e-1. The retarder is a Fuji film RPT146G,
which is a quarterwave plate at 586.5 nm. The parameters used in the program for the
retarder are: thickness = 63 μm, no = 1.5841, and ne = 1.5865. The LC material used is
ZLI-5049-000. The LC’s material parameters are $K_{33}/K_{11} = 1.45$, $K_{33}/K_{22} = 2.4$, $\varepsilon_\parallel = 12.9$, $\varepsilon_\perp = 4.5$, $n_o = 1.7152$, $n_o = 1.5075$, $d/p = 0.486$, $d = 3.98 \mu m$, and the twist angle is
220° (LH twist). The hologram is a DuPont green volume hologram. The hologram
material parameters used in the calculation are $n_{\text{max}} = 1.575$, $n_{\text{min}} = 1.415$, $d = 15\mu m$,
572 nm Bragg wavelength, 14° layer tilt angle, and the dielectric profile used is a chirped,
clipped sine wave.

The experimental data in Fig. 4-9 is measured with a Photo Research PR-650
SpectraScan® SpectraColorimeter™. The light source is an Oriel Quartz Tungsten
Halogen 100 W lamp housed in an Oriel Series Q Convective Lamp Housing. The
measured data was normalized to the light source spectra by dividing the measured
reflectance by the reflectance from a white diffusing glass. The data was put into a
percent scale by dividing the data by the peak reflectance.

The calculated data in Fig. 4-10 models the experimental data in Fig. 4-9 very
well. The full-width half-maximum (fwhm) of the measured data is 55 nm (spanning 525
nm to 580 nm) and the fwhm of the calculated data is 62 nm (spanning 523 nm to 585
nm). The measured data has a peak in the lower wavelength region, which is not seen in
calculated data. This is most likely due to that fact that the dielectric variation function is
a simple sine wave with an exponentially decreasing layer thickness. This peak at the
lower wavelengths could be simulated by creating, on average, more layers of a smaller
thickness that would correspond to the blue shift seen in the data. The calculated data has
a large tail at the higher wavelengths this is due to the exponential not being sharp enough, there are too many “thicker” layers. Changing B to larger values decreases the tail of the spectra but then the fwhm decreases.
FIG. 4-5. Chirped hologram spectra. The grating periodicity changes as a function of the thickness of the hologram. $A = 0.9$, the periodicity of the hologram has increased by approximately 1.1 times on one side of the hologram. The grating periodicity is 0.2 µm on the unchirped side of the hologram, the total grating thickness is 10.4 µm, $(n_2-n_1) = 0.059$, $n_1 = 1.4783$, $n_2 = 1.5217$, $\lambda_b = 514$ nm, 2000 layers. The light source has a 2 nm linewidth.
FIG. 4-6. Chirped hologram spectra. The grating periodicity changes as a function of the thickness of the hologram. $A = 0.8$, the periodicity of the hologram is increased by approximately 1.25 times on one side of the hologram. The grating periodicity is 0.2 $\mu$m on the unchirped side of the hologram, the total grating thickness is 10.4 $\mu$m, $(n_2-n_1) = 0.059$, $n_1 = 1.4783$, $n_2 = 1.5217$, $\lambda_b = 514$ nm, 2000 layers. The light source has a 2 nm linewidth.
FIG. 4-7. Chirped hologram spectra. The grating periodicity changes as a function of the thickness of the hologram. A = 0.7, the periodicity of the hologram is increased by approximately 1.45 times on one side of the hologram. The grating periodicity is 0.2 μm on the unchirped side of the hologram, the total grating thickness is 10.4 μm, (n₂-n₁) = 0.059, n₁ = 1.4783, n₂ = 1.5217, λ₀ = 514 nm, 2000 layers. The light source has a 2 nm linewidth.
FIG. 4-8. Chirped hologram spectra. The grating periodicity changes as a function of the thickness of the hologram. When $A = 0.6$, the periodicity of the hologram is increased by approximately 1.7 times on one side of the hologram. The grating periodicity is $0.2 \, \mu m$ on the unchirped side of the hologram, the total grating thickness is $10.4 \, \mu m$, $(n_2 - n_1) = 0.059$, $n_1 = 1.4783$, $n_2 = 1.5217$, $\lambda_b = 514 \, nm$, 2000 layers. The light source has a 2 nm linewidth.
FIG. 4-9. Experimental data. (A) Optical stack contains only the hologram. (B) Optical stack contains: polarizer, quarterwave, LC, and hologram, LC at 0 volts. Light is incident at 35°.
FIG. 4-10. Calculated data. (A) Optical stack contains only the hologram. (B) Optical stack contains: polarizer, quarterwave, LC, and hologram, LC at 0 volts. Light is incident at 35°.
4.2 Birefringent Holographic Layers

All other holograms modeled in this dissertation have isotropic dielectric layers. It is possible that the hologram's layers could consist of birefringent material instead of isotropic material. Birefringence is incorporated into the holographic layers by inputting not only one sine wave index profile but two different sine wave index profiles, one for the ordinary refractive index and one for the extraordinary refractive index. This is of interest to understand what effect this variation in the holographic layer will have on the spectra, to better understand whether or not the actual holograms investigated have isotropic or birefringent layers. Also, investigating this phenomena may lead to useful optical properties. The following graphs model birefringent hologram layers with the birefringence either in the plane of the hologram layers or out of the plane of the hologram layers.

As seen in Figs. 4-11 - 4-14, when the birefringence is in the plane of the hologram there are two peaks in the graph. The peak that is centered on 514 nm is due to reflection from the ordinary index, this is the s-polarization reflection. The peak that is centered either greater or less than 514 nm is due to reflection from the extraordinary index, this is the p-polarization reflection. The extraordinary index peak is centered on a wavelength greater than 514 nm (Fig. 4-11) if the layers were designed with positive birefringence, \( n_e = n_o + \Delta n \), and less than 514 nm (Fig. 4-12) if the layers were designed with negative birefringence, \( n_e = n_o - \Delta n \). For the larger in-plane birefringence, \( \Delta n = \pm 0.5 \), (Figs. 4-13 and 4-14) the second extraordinary peak must be shown on a second graph due to the large wavelength difference between the two peaks. For out-of-plane birefringence (Fig. 4-15...
there is only a small change seen in the spectra when comparing with isotropic hologram layers, a widening at the base of the spectra is seen for positive and negative birefringence.

When comparing actual hologram spectra with the generated hologram spectra in Figs. 4-11 through 4-18, it is determined that the photopolymer holograms we have investigated have isotropic layers. However, the simulated holograms with birefringent layers have interesting optical properties. When the birefringence is in the plane of the layers, there is a splitting of the reflectance peak that could be used to create reflective holograms for new applications that require polarization selection depending on wavelength.
FIG. 4-11. Birefringent hologram spectra. The birefringence is in the plane of the hologram and the optic axis is perpendicular to the hologram vertical axis. The grating periodicity is 0.2 μm, the total grating thickness is 10.4 μm, \((n_2-n_1) = 0.059\), \(n_1 = 1.4783\), \(n_2 = 1.5217\) (corresponding to ordinary index), \(\lambda_b = 514\) nm, 2000 layers. The light source has a 2 nm linewidth. Positive birefringence, \(n_e = n_0 + 0.1\).
FIG. 4-12. Birefringent hologram spectra. The birefringence is in the plane of the hologram and the optic axis is perpendicular to the hologram vertical axis. The grating periodicity is 0.2 μm, the total grating thickness is 10.4 μm, (n_2-n_1) = 0.059, n_1 = 1.4783, n_2 = 1.5217 (corresponding to ordinary index), λ_b = 514 nm, 2000 layers. The light source has a 2 nm linewidth. Negative birefringence, ne = no-0.1.
FIG. 4-13. Birefringent hologram spectra. The birefringence is in the plane of the hologram and the optic axis is perpendicular to the hologram vertical axis. The grating periodicity is 0.2 µm, the total grating thickness is 10.4 µm, \((n_2-n_1) = 0.059\), \(n_1 = 1.4783\), \(n_2 = 1.5217\) (corresponding to ordinary index), \(\lambda_b = 514\) nm, 2000 layers. The light source has a 2 nm linewidth. Positive birefringence, \(n_e = n_o + 0.5\), (a) ordinary peak, (b) extraordinary peak.
FIG. 4-14. Birefringent hologram spectra. The birefringence is in the plane of the hologram and the optic axis is perpendicular to the hologram vertical axis. The grating periodicity is 0.2 μm, the total grating thickness is 10.4 μm, \((n_2-n_1)=0.059\), \(n_1=1.4783\), \(n_2=1.5217\) (corresponding to ordinary index), \(λ_b=514\) nm, 2000 layers. The light source has a 2 nm linewidth. Negative birefringence, \(n_e=\text{no}-0.5\), (a) ordinary peak, (b) extraordinary peak.
FIG. 4-15. Birefringent hologram spectra. The birefringence is out of the plane of the hologram. The grating periodicity is 0.2 \( \mu \text{m} \), the total grating thickness is 10.4 \( \mu \text{m} \), \((n_2-n_1) = 0.059\), \(n_1 = 1.4783\), \(n_2 = 1.5217\) (corresponding to ordinary index), \(\lambda_b = 514\) nm, 2000 layers. The light source has a 2 nm linewidth. Positive birefringence, \(n_e = n_o + 0.1\).
FIG. 4-16. Birefringent hologram spectra. The birefringence is out of the plane of the hologram. The grating periodicity is 0.2 \( \mu \text{m} \), the total grating thickness is 10.4 \( \mu \text{m} \), \((n_2-n_1) = 0.059\), \(n_1 = 1.4783\), \(n_2 = 1.5217\) (corresponding to ordinary index), \(\lambda_h = 514\ \text{nm}\), 2000 layers. The light source has a 2 nm linewidth. Negative birefringence, \(n_e = n_o - 0.1\).
FIG. 4-17. Birefringent hologram spectra. The birefringence is out of the plane of the hologram. The grating periodicity is 0.2 μm, the total grating thickness is 10.4 μm, \((n_2 - n_1) = 0.059\), \(n_1 = 1.4783\), \(n_2 = 1.5217\) (corresponding to ordinary index), \(\lambda_b = 514\) nm, 2000 layers. The light source has a 2 nm linewidth. Positive birefringence, \(n_e = n_0 + 0.5\).
FIG. 4-18. Birefringent hologram spectra. The birefringence is out of the plane of the hologram. The grating periodicity is 0.2 μm, the total grating thickness is 10.4 μm, \( (n_2 - n_1) = 0.059 \), \( n_1 = 1.4783 \), \( n_2 = 1.5217 \) (corresponding to ordinary index), \( \lambda_b = 514 \) nm, 2000 layers. The light source has a 2 nm linewidth. Negative birefringence, \( n_e = n_o - 0.5 \).
CHAPTER 5

APPLICATIONS OF HOLOGRAPHIC REFLECTORS

5.1 Low Twist Single Polarizer Displays

A Reflective LCD using a holographic slanted reflector is optimized for the off axis angle of the holographic slanted reflector using a Berreman 4x4 simulation. The desired display configuration includes a polarizer, quarterwave plate, a liquid crystal cell, and a holographic reflector. The quarterwave plate is present so that the display can be operated in the “Normally White” mode, zero volts for the bright state and a higher voltage for the dark state. The “LCD Modeling” program is used to find the optimum LC twist angle, LC cell retardation, and rotation angles of the components in the display for the desired zero voltage bright state and five voltage dark state. The proposed display will not only provide a high contrast ratio but will include fewer components (the holographic reflector takes the place of the mirror, diffuser, and color filters) and does not have specular glare.

To find the best reflective configuration we created contour plots with the reflectance as the height of the contours and the x and y axes as retardation versus LC twist angle. This technique has been used by others, such as Kwok in ref 20. We investigate a direct view “Normally White” display and want to optimize for a very achromatic black state; therefore, our configuration contains one polarizer and a quarterwave plate. This situation is fundamentally different from crossed or parallel
polarizers because after the light goes through the polarizer and quarterwave plate it reaches the LC as circularly not linearly polarized light. This situation was not analyzed by Kwok but was reviewed by Kuo$^{34}$. We compare our simulated data below with Kuo’s even though the two sets of data were calculated using different LC materials. Using the program “LCD Modeling” several reflectance contour graphs, which include the entire parameter space, are generated to find the best configuration.

The holographic slanted reflector contour plots are generated for several different display configurations that are varied by rotating the individual components of the display. The rotated components are the polarizer (the quarterwave plate is always held at 45° to the polarizer transmission axis) and the LC cell. The display’s voltage is either 0 V or 5 V. The liquid crystal modeled is ZLI-4792, the parameters are listed as follows: $K_{11} = 13.2*10^{-12}$ N, $K_{22} = 6.5*10^{-12}$ N, $K_{33} = 18.3*10^{-12}$ N, $\varepsilon_{\perp} = 3.1$, $\Delta \varepsilon = +5.2$, $\Delta n = +0.0969$. All contour plots are calculated at 514 nm, the Bragg wavelength of the modeled hologram. In the contour plots the LC twist angle is varied every ten degrees along the x-axis and the retardation increases in steps of 0.02 μm by varying the thickness of the cell. This data is calculated with an ideal polarizer, which has the following parameters: $n_e = 1.53+i*0.5$, $n_o = 1.506$, $d = 3.3$ μm. The holographic slanted reflector plots are modeled with a holographic reflector that has an isotropic, sinusoidally varying index profile, the highest and lowest indices of that profile are, $n_1 = 1.4783$, and $n_2 = 1.5217$. The wavelength of the sinusoidal index profile satisfies the Bragg condition and has the following parameters: the Bragg wavelength is equal to 514 nm, the total
hologram thickness 10.4 μm, and the hologram is slanted at 14° (this is angle h in Fig. A-3 (b)).

The contour plots for the display containing a flat mirror only change very slightly when the polarizer or LC are rotated; therefore, the flat mirror display plots shown are representative of all polarizer rotations. The plot in Fig. 5-1 changes less then ten percent in retardation when the polarizer is rotated, this is illustrated in Fig. 5-2. This is expected because the light incident on the liquid crystal layer would be circularly polarized if the quarterwave plate was designed at the calculating wavelength, instead the light is slightly elliptical. The flat mirror plots are modeled with a mirror which has the following parameters: ne = 0.69+i*5.06, no = 0.69+i*5.06, d = 1 μm.

Since the holographic slanted reflector can also be rotated, there are several different plots for an LCD containing that component. The plots presented are those in which the optimum holographic reflector orientation has been chosen by maximizing the bright and minimizing the dark state. The optimum orientation of the holographic slanted reflector is found by rotating the LC cell through 180°, where the rotation angle is defined as the distance between the LC’s rub direction and the projection of the vertical reflecting axis of the hologram, and the polarizer is rotated through 45°, where the rotation angle is defined as the distance between the polarizer’s transmission axis and the input rub direction of the LC cell. Both rotations are in steps of 15°. Since this doesn’t cover every angle of the parameter space other possible configurations may exist.
FIG. 5-1. Reflectance contour plot for bright state (0 volts) of an LCD which contains a flat mirror. The polarizer is aligned at 0°. The contours indicate constant reflectance which takes the following steps 0.1, 0.2, 0.3, 0.35, 0.4, and 0.45.
FIG. 5-2. Reflectance contour plot for bright state (0 volts) of an LCD which contains a flat mirror. The polarizer is aligned at 30°.
Figs. 5-1 and 5-3 show the reflectance contour plots for the bright state of a display containing a mirror. In this case the height of the contour lines is the value of reflectance for the corresponding LC twist angle and retardation. Fig. 5-3 shows the results from these simulations along with the data points published in Ref. Ref. It is found, in both the simulated data and Ref. Ref., that LCDs which have twists of 70°, 80°, and 90° and retardation values of 0.275, 0.26, and 0.24 μm, respectively, have decreasing brightness for the bright state. The simulated data finds the following transmissions for the bright state: $T_{70} \approx 0.43$, $T_{80} \approx 0.41$, and $T_{90} \approx 0.36$, where parallel polarizers have a transmission of 0.5. Ref. Ref. finds the following transmissions for the bright state: $T_{70} \approx 73$, $T_{80} \approx 72$, and $T_{90} \approx 70$, where parallel polarizers has a transmission of 76. Figs. 5-4 and 5-5 show the reflectance contour plots for the dark state of a display containing a mirror. Once again, the resulting data from these simulations is compared with the data published in Ref. Ref. It is found that LC cells which have twists of 70°, 80°, and 90° and retardation values of 0.275, 0.26, and 0.24 μm, respectively, have decreasing darkness for the dark state. The simulated data finds the following transmissions for the dark state: $T_{70} \approx 0.03$, $T_{80} \approx 0.01$, and $T_{90} \approx 0.003$. Ref. Ref. finds the following transmissions for the dark state: $T_{70} \approx 5$, $T_{80} \approx 2$, and $T_{90} \approx 1$. The best dark state for an LCD containing a flat mirror can be obtained with a 100° twist LCD since it has the least reflectance. However, the bright state of the 100° twist LCD is not nearly as good as that of the 70°, 80°, or 90°, therefore the 100° twist cell is not normally used for displays.
FIG. 5-3. Enlarged region of interest for the reflectance contour plot from Figure 5-1.

When this plot is compared with previous published data \(^{34}\), it is found that LCDs which
have twists of 70°, 80°, and 90° and \(\Delta n\)d values of 0.275, 0.26, and 0.24 \(\mu m\),
respectively, have decreasing brightness which is seen above, marked by dots.
FIG. 5-4. Reflectance contour plot for dark state (5 volts) of an LCD which contains a flat mirror. The contours indicate constant reflectance which takes the following steps 0.0001, 0.001, 0.005, 0.01, 0.05, 0.1, and 0.5.
FIG. 5-5. Enlarged region of interest for the reflectance contour plot from Figure 5-3. When this plot is compared with previous published data $^{34}$, it is found that LC cells which have twists of 70°, 80°, and 90° and Δn values of 0.275, 0.26, and 0.24 μm, respectively, have decreasing darkness which is seen above, marked by dots.
Fig. 5-6 shows the reflectance contour plot for the bright state of a display containing a HSR. The optimum orientation of the holographic slanted reflector was found to have the input rub direction of the LC cell rotated $0^\circ$ from the projection of the vertical reflecting axis of the hologram and the polarizer's transmission axis rotated $0^\circ$ from the input rub direction of the LC cell. The maximum brightness of a display containing an HSR is 35% while the maximum brightness a display containing a mirror is 40% where the scale is normalized to parallel polarizers at 50%. Fig. 5-7 shows the reflectance contour plot for the dark state of a display containing a HSR. The darkest a display containing an HSR can get is 0.01% while the display containing a mirror can only get as dark as 0.1%. Therefore, the display containing a mirror can achieve a brighter bright state but the display containing a HSR can achieve a much darker dark state.

As can be seen from the dark and bright state plots, an optimized display with a holographic slanted reflector is predicted to have good dark and bright states when the LC twist angle is $80^\circ$ and the retardation is between 0.2 $\mu$m and 0.45 $\mu$m. The optimized flat mirror display has good dark and bright states when the LC twist angle is $100^\circ$ and the retardation is between 0 and 0.3 $\mu$m. Our own simulation data along with Kuo's calculated data was compared with simulation data taken using the Berreman program written by Dwight Berreman and good agreement was achieved with each set of data using its respective LC parameters.
FIG. 5-6. Reflectance contour plot for bright state (0 volts) of an LCD which contains a holographic slanted reflector. The contours indicate constant reflectance which takes the following steps 0.1, 0.2, 0.3, 0.35, 0.4, and 0.45.
FIG. 5-7. Reflectance contour plot for dark state (5 volts) of an LCD which contains a holographic slanted reflector. The contours indicate constant reflectance which takes the following steps 0.0001, 0.001, 0.005, 0.01, 0.05, 0.1, and 0.5.
5.2 High Twist Single Polarizer Displays

Single polarizer reflective STN-LCDs have the advantages of low power consumption, good contrast ratio; and no parallax because the reflector is internal to the display. However, because the display is rather chromatic it needs to be compensated by birefringent films. Each component added to the display introduces several new variables making the optical design increasingly more complicated. The single polarizer reflective display contains a polarizer, stack of retarders, LC layer, and a reflector.

A display optimization method is devised that considers the variables of the LC layer without the added variables of the polarizer and retarders. This method assumes that the polarization input into the LC layer will be well defined and makes no assumptions about the polarization between the LC layer and the reflector. This assumption need not be adjusted for a holographic slanted reflector (HSR), and is more generally applicable then previous optimization methods.\textsuperscript{22, 23, 35, 36}

Once the polarizer and retarder variables are eliminated from the optimization the remaining variables are those of the LC layer. The variables for the LC layer are; bright state voltage ($V_{\text{bright}}$), dark state voltage ($V_{\text{dark}}$), and $\Delta n/\lambda$. The devised method is based on inputting some known polarization into the LC layer, calculating the light propagation through that layer and reflecting the light off the reflector. This calculation results in a polarization state exiting the LC layer (P6 Fig. 5-8) that is later compared with other polarization states in the display. The output of the program is converted to Stokes parameters and the polarization states are looked at in terms of Poincare sphere angles. In previous works,\textsuperscript{22, 23, 35, 36} the assumption is made that the optimized display will have
linearly polarized light between the LC layer and reflector, P4, for the bright state and circularly polarized light at P4 for the dark state. This method works well for a display containing a flat mirror but a display that contains an HSR can not have those assumptions made because of differing path lengths for the input and output light rays in the LC layer. This method will be implemented for a display containing a mirror and then for a display containing an HSR. The differences between the polarization states at the two different reflectors are also considered.
FIG. 5-8. Single polarizer optical stack. P1 through P8 label the polarization for that location in the optical stack \(^2^3\).
5.2.1 Optimizing a Display by Matching Polarization States

The most fundamental requirements for an LCD are considered. Several conditions exist that must be as closely fulfilled as possible for a good display, the conditions are as follows;

(1) For complete extinction of the display, the ideal dark state will have the polarization at P3 orthogonal to the polarization at P6.

(2) For high contrast ratio; the polarization at P6 for the dark state will be orthogonal to the polarization at P6 for the bright state.

(3) For complete reflection from the display, the ideal bright state will have the polarization at P3 exactly the same as the polarization at P6.

Conditions (2) and (3) are redundant. If conditions (1) and (3) are met, then condition (2) is automatically met and vice versa. When the dark state polarization P6 is orthogonal to the input polarization P3 and the bright state polarization P6 is parallel to the input polarization P3, then the dark state polarization P6 must be orthogonal to the bright state polarization P6. For a display containing a flat mirror to be bright, condition (3), the polarization of light reflected at the mirror must not change polarization states (P4 = P5). For on-axis light, the polarization at the mirror experiences a $\pi$-phase shift, leaving linear polarizations unaltered upon reflection and circular polarizations undergoing an orthogonal change in polarization. Therefore, the polarization at the mirror (P4) for the bright state must be linear and for the dark state must be circularly polarized, as is the assumption indicated in previous methods $^{22, 23, 35, 36}$. These conclusions depend on the retarder stack not changing the relative difference between the
polarization states on the entrance and exiting light ray. In other words, the relative angular distance between the polarization states P3 and P6 must be equal to the relative angular distance between the polarization states P2 and P7 along with P1 and P8. This is proved in Appendix C.

In the devised method it is assumed that there is some well defined polarization incident on the display, at P3, created by a polarizer and stack of angular independent retarders. At this time the exact contents of that stack are not considered, that will be found once the LC layer’s variables and hologram’s orientation are defined. Instead, the devised method focuses on finding the LC layer which will most closely fulfill the conditions stated above. One other existing restriction is that the polarization at P3 will be equal for both the bright and dark states; therefore, allowing the program to cycle through several input polarization’s at P3. The input polarization state vector is defined on the Poincare sphere using Stokes parameters. The polar angle, \(2\omega\), is initially incremented in steps of 10° from 0° to 180° and the azimuthal angle, \(2\alpha\), is initially incremented in steps of 10° from 0° to 360°, see Fig. 5-9. Once the input polarization is defined using Stokes parameters it is converted to Jones representation, sent into the Berreman 4x4 method and propagated through the LC layer. Since the STN typically has greater chromaticity issues in the dark state, the optimum variables for that state will be found first and then the optimum variables for the bright state will be found.

For the display containing a holographic slanted reflector there is one additional variable, the angle between the LC rub direction and the hologram’s viewing axis. This angle is incremented in steps of 20° from 0° to 360° for this simulation.
FIG. 5-9. Angular definitions on the Poincare Sphere. The angles $\gamma_{\text{dark}}$, $2\omega_{P6}$, and $2\alpha_{P6}$ are the polar and azimuthal angles that describe the orientation of the polarization state P6 on the Poincare sphere. $\gamma_{\text{dark}}$ is the angular distance between the spolarization states P3 and P6.
5.2.2 Applying the Optimization Method

The LC material used for this simulation is ZLI-5049-000. The LC’s material parameters are $K_{33}/K_{11} = 1.45$, $K_{33}/K_{22} = 2.4$, $\varepsilon_{||} = 12.9$, $\varepsilon_{\perp} = 4.5$, $n_e = 1.7152$, $n_o = 1.5075$, $d/p = 0.486$, and the twist angle is chosen to be $220^\circ$ (LH twist). The input rub direction of the LC layer is defined as $2\alpha = 0$, unless otherwise stated. The mirror’s material parameters are $n = 0.69+i*5.06$ and $d = 1\mu m$. The hologram’s material parameters are $n_{\text{max}} = 1.5327$, $n_{\text{min}} = 1.4673$, $d = 15\mu m$, 565 nm Bragg wavelength, 14° layer tilt angle, and the dielectric profile used is a clipped sine wave.

All calculations are completed at 550 nm for the mirror configuration and 558 nm for the HSR configuration. The incident angle is 0° for the mirror configuration and 35° for the HSR configuration. The dark state voltage is defined as $V_{\text{dark}} = 2.0$ V and the bright state voltage is defined as $V_{\text{bright}} = 1.73$ V. These voltages were defined by locating the span of voltages which have the largest mid-layer director change, i.e. the largest slope on a plot of mid-layer theta versus voltage. The mid-layer theta value for $V_{\text{dark}}$ is 40° and the mid-layer theta value for $V_{\text{bright}}$ is 80°. These mid-layer theta values are the same as those used in Ref. 23. The axes used in Ref. 23 are 90° different then what is used here; therefore, the values described as 40° and 80° are described as 50° and 10°, respectively. The remaining variables to be solved for are the input polarization at P3 and $\Delta n d/\lambda$.

The light propagation through the LC in the dark state is considered first. For the purposes of this simulation light propagation begins with the input polarization at P3. The input polarization, P3, is propagated through the LC layer and then reflected off the
reflector and back through the LC layer ending with a value for the polarization at P6\textsubscript{dark}. This simple calculation is done for each input polarization state at P3, as described above, and varies $\Delta n d/\lambda$. The output polarization at P6\textsubscript{dark} is converted back into Stokes parameters and compared with the input polarization P3 via the angular distance, $\gamma_{\text{dark}}$, between those two polarizations on the Poincare sphere.

$$
\gamma_{\text{dark}} = \cos^{-1} \left[ \cos(2\omega_{P3}) \cos(2\omega_{P6\text{dark}}) \\
+ \sin(2\omega_{P3}) \sin(2\omega_{P6\text{dark}}) \cos(2\alpha_{P3} - 2\alpha_{P6\text{dark}}) \right] 
$$

where $2\omega_{P3}$ is the input polarization’s polar angle, $2\omega_{P6\text{dark}}$ is the output polarization’s polar angle for the dark state, $2\alpha_{P3}$ is the input polarization’s azimuthal angle, $2\alpha_{P6\text{dark}}$ is the output polarization’s azimuthal angle for the dark state. Because Poincare sphere angles are two times lab angles, an orthogonal angular distance will be 180° and a parallel angular distance is 0°.

Once $\gamma_{\text{dark}}$ is found, a second calculation takes place in which light is propagated through the LC in the bright state. The same set of P3’s are propagated through the LC layer, now in the bright state, and then reflected off the reflector and back through the LC layer ending with a value for the polarization at P6\textsubscript{bright}. A solution for P3 and $\Delta n d/\lambda$ can be found in one of two ways; calculating the angular distance between P3 and P6\textsubscript{bright} for the bright state, $\gamma_{\text{bright}}$, and comparing with $\gamma_{\text{dark}}$ until a $\Delta n d/\lambda$ exists where $\gamma_{\text{bright}}$ is zero degrees and $\gamma_{\text{dark}}$ is 180°, or calculating the angular distance between P6\textsubscript{dark} and P6\textsubscript{bright}, $\gamma_{\text{d,b}}$, and finding the $\Delta n d/\lambda$ at which $\gamma_{\text{d,b}}$ is 180° and $\gamma_{\text{dark}}$ is 180°. Angular distances are defined as:
\[ \gamma_{\text{bright}} = \cos^{-1} \left[ \cos(2\omega_p) \cdot \cos(2\omega_{p6\text{ bright}}) + \sin(2\omega_p) \cdot \sin(2\omega_{p6\text{ bright}}) \cdot \cos(2\alpha_p - 2\alpha_{p6\text{ bright}}) \right] \] (5.2)

and,

\[ \gamma_{d,b} = \cos^{-1} \left[ \cos(2\omega_{p6\text{ dark}}) \cdot \cos(2\omega_{p6\text{ bright}}) + \sin(2\omega_{p6\text{ dark}}) \cdot \sin(2\omega_{p6\text{ bright}}) \cdot \cos(2\alpha_{p6\text{ dark}} - 2\alpha_{p6\text{ bright}}) \right] \] (5.3)

where \(2\omega_{p6\text{ bright}}\) is the output polarization's polar angle for the bright state, \(2\alpha_{p6\text{ bright}}\) is the output polarization's azimuthal angle for the bright state, and the other angles were previously defined. Both the dark and bright angular distances, \(\gamma_{\text{dark}}\) and \(\gamma_{\text{bright}}\), are plotted versus \(\Delta n/\lambda\) in Figs. 5-10 and 5-11 for a fixed P3. Figs. 5-12 through 5-13 show the dependence \(\gamma_{\text{dark}}\) and \(\gamma_{\text{bright}}\) has on the input polar and azimuthal angles respectively (fixed \(\Delta n/\lambda\)).

The variables found to work best for the display containing a flat mirror are: \(\Delta n/\lambda = 1.427\), \(2\omega = 72.7^\circ\), and \(2\alpha = 187.2^\circ\), and for the display containing a slanted holographic reflector are: \(\Delta n/\lambda = 1.481\), \(2\omega = 118^\circ\), and \(2\alpha = 186^\circ\) with the LC input rub rotated to \(260^\circ\). The mirror result compares well with published data. In Ref. 22 a 220° twist cell with the same dark and bright voltage mid-layer thetas presented here finds a solution where \(\Delta n/\lambda \approx 1.4\) (read off graph). There is not previous published data for a single polarizer STN containing a slanted holographic reflector.

The devised method allows one to determine the LC variables independent of the polarizer and retarder variables, thus decreasing the number of variable permutations. The input polarization is assumed to be well defined at P3 and is propagated through the
LC layer resulting in solutions for the LC variables. For the mirror configuration, the polarization at P4 for the bright state is linearly polarized and the polarization at P4 for the dark state is circularly polarized as indicated in previous works\textsuperscript{22, 23, 35, 36}.

The devised method is expanded from previous methods to include a Holographic Slanted Reflector instead of a mirror. Because the input and output angles of the HSR are not equal, the polarization at the reflector is not exactly linear or circularly polarized, as is the case for the flat mirror. Because of the elliptical polarization at the reflector previous methods could not have been used to solve for the STN-LCD configuration that contains an HSR. The solution presented for the HSR configuration is not the only correct one. If the LC input rub direction is rotated and $\Delta nd/\lambda$ is varied, solutions of a similar caliber can be found.
FIG. 5-10. Angular distance, $\gamma_{\text{dark}}$ and $\gamma_{\text{bright}}$, between P3 and P6 versus $\Delta nd/\lambda$ for $V_{\text{dark}} = 2.0$ V (solid) and $V_{\text{bright}} = 1.73$ V (dotted). 220° twist cell. Stack containing mirror: input polarization’s angles $2\omega = 72.7^\circ$ and $2\alpha = 187.2^\circ$. 

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FIG. 5-11. Angular distance, $\gamma_{\text{dark}}$ and $\gamma_{\text{bright}}$, between P3 and P6 versus $\Delta n d / \lambda$ for $V_{\text{dark}} = 2.0$ V (solid) and $V_{\text{bright}} = 1.73$ V (dotted). 220° twist cell. Stack containing holographic slanted reflector: LC input rub at 260°, input polarization’s angles $2\omega = 118^\circ$ and $2\alpha = 186^\circ$. 

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FIG. 5-12. This plots the angular distance, $\gamma_{\text{dark}}$ and $\gamma_{\text{bright}}$, between P3 and P6 versus the input polarization angle $2\alpha$, for $V_{\text{dark}} = 2.0$ V (solid) and $V_{\text{bright}} = 1.73$ V (dotted). 220° twist cell. Stack containing mirror: the input polarization’s azimuthal angle $2\alpha = 187.2^\circ$ and $\Delta n d/\lambda = 1.427$. 
FIG. 5-13. This plots the angular distance, $\gamma_{\text{dark}}$ and $\gamma_{\text{bright}}$, between P3 and P6 versus the input polarization angle $2\alpha$, for $V_{\text{dark}} = 2.0$ V (solid) and $V_{\text{bright}} = 1.73$ V (dotted). 220° twist cell. Stack containing holographic slanted reflector: the input polarization’s azimuthal angle $2\alpha = 186^\circ$ and $\Delta n/d = 1.481$. 

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FIG. 5-14. This plots the angular distance, $\gamma_{\text{dark}}$ and $\gamma_{\text{bright}}$, between P3 and P6 versus the input polarization angle $2\alpha$, for $V_{\text{dark}} = 2.0 \text{ V}$ (solid) and $V_{\text{bright}} = 1.73 \text{ V}$ (dotted). 220° twist cell. Stack containing mirror: input polarization’s polar angle $2\omega = 72.7°$ and $\Delta n d/\lambda = 1.427$. 

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FIG. 5- 15. This plots the angular distance, \( \gamma_{\text{dark}} \) and \( \gamma_{\text{bright}} \), between P3 and P6 versus the input polarization angle \( 2\alpha \), for \( V_{\text{dark}} = 2.0 \) V (solid) and \( V_{\text{bright}} = 1.73 \) V (dotted). 220° twist cell. Stack containing holographic slanted reflector: input polarization’s polar angle \( 2\omega = 118° \) and \( \Delta n d/\lambda = 1.481 \).
5.2.3 Finding the Polarizer and Retarder Stack

The polarizer and retarder stack can be defined after the correct polarization state at P3 is found. The retardation of the retarder, \( \Delta n d_R \), is limited by the number of commercially available retarders. For this example, the retarder is a quarterwave plate at 586.5 nm. This retarder is picked for several reasons; it is readily available in the lab, it's a uniaxial retarder (therefore easily modeled), a quarterwave plate can rotate an initial polarization to any point on the Poincare sphere at it's designed wavelength, and 586.5 nm is fairly close to the wavelength of interest, 550 nm. If an achromatic retarder stack is desired then several retarders will be incorporated into the stack, instead of just one. The standard method of generating an achromatic quarterwave plate is to stack one quarterwave plate with one or two halfwave plates. The axes orientation of each retarder varies depending on the desired wavelength dependence of the retarder stack. This theory has been reviewed by several authors starting as far back as 1955 \(^{12, 24, 38}\). The following data is calculated using parameters found for the Fuji RPT 146G quarterwave plate, those parameters are: \( n_e = 1.5865 \), \( n_o = 1.5841 \), \( d = 63 \mu m \). The calculation parameters for the ideal polarizer are: \( n_e = 1.53+i*0.5 \), \( n_o = 1.51+i*0.004 \), \( d = 3.3 \mu m \).

To find the correct polarizer and retarder angles, the light propagation calculation begins at the polarization state outside of the polarizer, prior to P1 (Fig. 5-8). The calculation starts with circularly polarized light incident on the polarizer and continues through P1 and the retarder (in this case there is a single retarder and; therefore, no P2) and ends at P3. Ideally, unpolarized light would be incident on the device but since there is no way for Jones or Berreman vectors to represent unpolarized light, circularly
polarized light is used instead. This is an accurate approximation to unpolarized light in the case where the first optical element that the light sees is a polarizer, which creates a well defined polarization state. The polarizer and retarder are rotated generating a set of output polarizations, $P_{3_{test}}$. The angular distance between $P_{3_{test}}$ and the desired polarization, $P_3$ (defined from the optimization in section 5.2.2), is $\gamma_{PR}$. The minimum angular distance, $\gamma_{PR}$, locates the polarizer and retarder angles. This minima is found using the contour plots in Figs. 5-16 and 5-17 where the height of the contour plots is the angular distance $\gamma_{PR}$, "Rotation 1 angle" is the polarizer's rotation angle, and "Rotation 2 angle" is the quarterwave plate's rotation angle.

For the display containing a mirror the desired output polarization ($P_3$) is $2\omega = 72.7^\circ$ and $2\alpha = 187.2^\circ$. Using Fig. 5-16 the minimum angular distance, $\gamma_{PR}$, is found when the polarizer's optic axis azimuthal angle is $16^\circ$ and the retarder's optic axis azimuthal angle is $6^\circ$. For the display containing a HSR the desired output polarization ($P_3$) is $2\omega = 118^\circ$ and $2\alpha = 186^\circ$. Using Fig. 5-17 the minimum angular distance, $\gamma_{PR}$, is found when the polarizer's optic axis azimuthal angle is $19^\circ$ and the retarder's optic axis azimuthal angle is $94^\circ$. 
FIG. 5-16. Contour plot of Retarder rotation angle (Rotation 2) vs. Polarizer rotation angle (Rotation 1), where the height of the contours is the angular distance $\gamma_{PR}$. The contour values are $20^\circ$, $10^\circ$, $2^\circ$, $1^\circ$. This plot is for the stack containing a mirror; therefore, the desired output polarization is $2\omega = 72.7^\circ$ and $2\alpha = 187.2^\circ$. From this plot the polarizer angle is chosen to be $16^\circ$ and the retarder angle is chosen to be $6^\circ$. 
FIG. 5-17. Contour plot of Retarder rotation angle (Rotation 2) vs. Polarizer rotation angle (Rotation 1), where the height of the contours is the angular distance $\gamma_{PR}$. The contour values are $20^\circ, 10^\circ, 2^\circ, 1^\circ$. This plot is for the stack containing a hologram, therefore, the desired output polarization is $2\omega = 118^\circ$ and $2\alpha = 186^\circ$. From this plot the polarizer angle is chosen to be $19^\circ$ and the retarder angle is chosen to be $94^\circ$. 
FIG. 5-18. Reflection versus wavelength for the mirror configuration, containing the polarizer retarder stack found in Fig. 5.16. a) Non-select voltage 1.73 V. b) Select voltage 2.0 V.
FIG. 5-19. Reflection versus wavelength for the hologram configuration, containing the polarizer retarder stack found in Fig. 5.17. a) Non-select voltage 1.73 V. b) Select voltage 2.0 V.
CHAPTER 6

SUMMARY AND CONCLUSION

6.1 Form Birefringent Holograms

6.1.1 Hologram Properties

In the form birefringent regime, the theoretical predicted values for the birefringence agrees well with the birefringence calculated using the Berreman method. When comparing dielectric profiles, the square wave dielectric profile has the largest average birefringence for the same material parameters. The hologram’s birefringence varies in the same way as a linear retarder when the incident angle and total thickness are varied. The dispersion of the hologram can be tuned by varying its periodicity, until the Bragg regime is reached. The hologram’s dispersion can be matched to a LC materials allowing for very good compensation, better than the currently available compensators. More work could be done to find the ideal hologram dispersion at all angles by varying the hologram index values, which would require investigating new photopolymer holographic materials with larger differences between the minimum and maximum index values.

6.1.2 Applications of Holographic Retarders

Dispersion matched retarders compensate the LC layer more completely then retarders with a different dispersion curve. Negative birefringence dispersion matched retarders can be generated using form birefringent holograms by varying the periodicity
of the index profile. Dispersion matched retarders could be used for several display modes such as the two considered here; VA and TN.

Using a stack of retarders versus a single splayed retarder has the advantage of not only being able to more accurately match the LC’s optic axis polar angle but also allows the azimuthal angle to be matched by rotating each retarder. Retarders that have their wavelength dispersion matched to the LC’s compensate the dark state more accurately. All of the compensated displays, except the display compensated by the 1 layer retarder, improved the dark state of the uncompensated TN for all wavelengths, which resulted in an improved viewing cone. The bright state of the uncompensated TN was the brightest display shown but the decrease in brightness for the 3 layer ideal retarder, 3 layer hologram, and the 4 layer ideal retarder was minimal.

The most uniform viewing cones occurred for the displays compensated by the ideal retarder stacks because the path length inside those retarders is almost identical to the path length inside the LC. The viewing cone of the display compensated by the 3 layer hologram is somewhat asymmetric because of the difference in path length in the hologram versus that in the LC. This could be improved by matching the hologram’s ordinary and extraordinary indices more closely to the LC’s. Form birefringent holographic retarders have the added advantage of a variable wavelength dispersion that can be tuned to match the LC’s by varying the hologram’s periodicity. These retarders also allow for any desired uniform optic axis orientation.
6.2 Reflective Holograms

6.2.1 Hologram Properties

The Berreman 4x4 method is well suited to simulate different holographic index profiles, chirped gratings, and birefringent grating layers. This method allows one to vary several aspects of the hologram’s index profile easily and without need for a complete rederrivation of the entire theory. This versatility is useful when modeling photopolymeric holograms because of the numerous processing steps involved to create them. The required processing steps may alter the index profile’s sinusoidal shape that is defined during the initial laser exposure step; therefore, many of the effects investigated could be seen in holographic spectra.

6.2.2 Low Twist Displays Containing Holographic Reflectors

As can be seen from the contour plots in chapter 5, an optimized display with a holographic slanted reflector is predicted to have a good dark and bright state and a low LC twist angle. The optimized holographic slanted reflector display has a LC layer with an 80° twist and a retardation between 0.2 μm and 0.45 μm. The optimized flat mirror display has a LC layer with a 100° twist and a retardation between 0 and 0.3 μm. Our own simulation data along with Kuo’s calculated data was compared with simulation data taken using the Berreman program written by Dwight Berreman and good agreement was achieved with each set of data using its respective LC parameters. The proposed device will not only provide a high contrast ratio but will include fewer components (the holographic reflector takes the place of the mirror, diffuser, and color filters) and does not have specular glare.
6.2.3 High Twist Displays Containing Holographic Reflectors

A method is described that determines the variables of the LC layer for a single polarizer STN-LCD. This method allows one to determine the LC variables independent of the polarizer and retarder variables, thus decreasing the number of variable permutations. The input polarization is assumed to be well defined at P3 and is propagated through the LC layer resulting in solutions for the LC variables. For the mirror configuration, the polarization at P4 for the bright state is linearly polarized and the polarization at P4 for the dark state is circularly polarized as indicated in previous works \(^{22,23,35,36}\).

We have expanded this method to include a Holographic Slanted Reflector instead of a mirror. Because the input and output angles of the HSR are not equal, the polarization at the reflector is not exactly linear or circularly polarized, as is the case for the flat mirror. Because of the elliptical polarization at the reflector previous methods could not have been used to solve for the STN-LCD configuration that contains an HSR. The solution presented for the HSR configuration is not the only correct one. If the LC input rub direction is rotated and \(\Delta n/d\) is varied, solutions of a similar caliber can be found.

The variables found to work best for the display containing a flat mirror are: \(\Delta n/d = 1.427\), \(2\omega = 72.7^\circ\), and \(2\alpha = 187.2^\circ\), and for the display containing a slanted holographic reflector are: \(\Delta n/d = 1.481\), \(2\omega = 118^\circ\), and \(2\alpha = 186^\circ\) with the LC input rub rotated to \(260^\circ\). The mirror result compares well with published data. In Ref. \(^{23}\) a \(220^\circ\) twist cell with the same dark and bright voltage mid-layer thetas presented here
finds a solution where $\Delta n d/\lambda \approx 1.4$ (read off graph). There is not previous data for a single polarizer STN containing a slanted holographic reflector for comparison.
APPENDIX A

MODELING VOLUME HOLOGRAMS

A.1 Modeling Methods for Holograms

A.1.1 Standard Modeling Method; Coupled Wave Theory

The coupled wave theory is the standard method for modeling volume holograms. The definitions derived from this theory are used in the form birefringence equations presented in chapter 2. The theory makes several assumptions, the first is that monochromatic light is incident on the hologram, either at the Bragg angle or near it. This theory is derived for light polarized perpendicular or parallel to the plane of incidence but is not valid for arbitrary input polarizations. The final assumption is that the incoming reference wave and outgoing signal wave are the only two light waves in the hologram, neglecting higher orders. This assumption is valid because higher diffraction orders strongly violate the Bragg condition. Due to the last assumption, coupled wave theory is only valid for thick holograms, or holograms in which the incident light wave diminishes as it propagates through the hologram due to the high percentage of light diffracted. Each different dielectric profile described by the coupled wave theory requires the resulting electric field equations to be rederived; therefore, using the Berreman method to describe several dielectric profiles is much more straightforward.
The typical parameters used are shown in Fig. A-1. The fringe planes are the planes of dielectric variation in the hologram. The grating vector, $\vec{K}$, is defined to be perpendicular to the hologram’s fringe planes, and its length is $K = 2\pi/\Lambda$, where $\Lambda$ is the hologram’s periodicity.
FIG. A-1. Thick volume hologram. The index modulation or dielectric profile is demonstrated using dotted lines. The parameters shown are, periodicity ($\Lambda$), grating vector ($\vec{K}$), total hologram thickness ($d$), incident angle, ($\theta$), exiting angle ($\phi$), layer angle ($h$).
A.1.2 Berreman 4x4 Method

The 4x4 matrix method solves Maxwell's equations for anisotropic media with no free charges. The Berreman 4x4 method, one version of the 4x4 matrix method, calculates light propagation through a stack of anisotropic, thin, uniform dielectric layers that are parallel to the surface of the stack. It assumes that those layers are nonmagnetic, nonconducting, and electrically anisotropic. Introducing an imaginary index of refraction, allows the anisotropic layers to be optically absorbing. The dielectric permittivity tensor is constant in the y and z directions and only changes with respect to x, Fig. A-2. The Berreman method allows for an arbitrary dielectric profile along the thickness of the material. In the Berreman 4x4 program used for these calculations we include Matlab toolbox functions written by Hodgkinson and Wu.

The derivation of the 4x4 matrix method begins by solving Maxwell's equations for anisotropic media with no free charges,

\[
\begin{align*}
\nabla \times \vec{E} &= -\mu_o \frac{\partial \vec{H}}{\partial t} \quad \nabla \cdot \vec{D} = 0 \\
\nabla \times \vec{H} &= \varepsilon_o \varepsilon \frac{\partial \vec{E}}{\partial t} \quad \nabla \cdot \vec{H} = 0
\end{align*}
\]

(A.1)

where \(\vec{E}\) is the electric field, \(\vec{H}\) is the magnetic field, \(\vec{D}\) is the electric displacement, \(\mu_o\) is the permeability of vacuum, \(\varepsilon_o\) is the permittivity of vacuum, and \(\varepsilon\) is the dielectric permittivity tensor. The dielectric permittivity tensor is symmetric:

\[
\varepsilon = \begin{pmatrix}
\varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\
\varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\
\varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz}
\end{pmatrix}.
\]

(A.2)
FIG. A-2. Berreman discretization and axes definition. The dielectric permittivity tensor varies from layer to layer but within each layer it is constant. The azimuthal angle, \( \varphi \), (equal to zero above) starts at the positive \( y \)-axis and rotates in the positive direction to the positive \( z \)-axis.
The electric and magnetic field vectors are assumed to be plane waves of the form:

\[
\vec{E} = \vec{E}_0 e^{i(k \cdot \vec{r} - \omega t)} = \vec{E}(x) e^{i(k_x x - \omega t)}
\]

\[
\vec{H} = \vec{H}_0 e^{i(k \cdot \vec{r} - \omega t)} = \vec{H}(x) e^{i(k_x x - \omega t)},
\]

where \( \vec{r} \) is the displacement vector, \( \omega \) is the angular frequency of the waves, and \( \vec{k} \) is the wave vector that has a magnitude of \( k_0 = 2\pi/\lambda_0 \), and \( \lambda_0 \) is the wavelength of the plane wave in vacuum. The calculation is simplified by defining the wave vector in only the xy plane; any other orientation can be created by simply rotating the axes frame. Two of Maxwell’s equations are rewritten using the equations for \( \vec{E} \) and \( \vec{H} \), in Eq. (A.3).

\[
\begin{pmatrix}
0 & 0 & ik_y \\
0 & 0 & -\frac{d}{dx} \\
-ik_y & \frac{d}{dx} & 0
\end{pmatrix}
\begin{pmatrix}
E_x \\
E_y \\
E_z
\end{pmatrix}
= ik_0 \frac{\varepsilon_0}{z_0}
\begin{pmatrix}
H_x \\
H_y \\
H_z
\end{pmatrix}
\]  

(A.4)

\[
\begin{pmatrix}
0 & 0 & ik_y \\
0 & 0 & -\frac{d}{dx} \\
-ik_y & \frac{d}{dx} & 0
\end{pmatrix}
\begin{pmatrix}
H_x \\
H_y \\
H_z
\end{pmatrix}
= -i \frac{k_0}{z_0}
\begin{pmatrix}
\varepsilon_{xx} E_x + \varepsilon_{xy} E_y + \varepsilon_{xz} E_z \\
\varepsilon_{yx} E_x + \varepsilon_{yy} E_y + \varepsilon_{yz} E_z \\
\varepsilon_{zx} E_x + \varepsilon_{zy} E_y + \varepsilon_{zz} E_z
\end{pmatrix}
\]  

(A.5)

The constant, \( z_0 \), is the impedance of vacuum and is equal to \( \sqrt{\mu_0/\varepsilon_0} \). Eqs. (A.4) and (A.5) represent six equations in which two of those equations are independent of derivatives. The tangential components, \( E_y, H_y, E_z, \) and \( H_z \) are continuous across boundaries while the normal components, \( E_x \) and \( H_x \), may be discontinuous. Because
the normal components may be discontinuous they are eliminated from the rest of the
calculation by using the two equations that do not contain derivatives, found in the sets of
equations in (A.4) and (A.5).

\[ H_x = \frac{\beta}{z_o} E_z \quad (A.6), \]

\[ E_x = -\frac{1}{\varepsilon_{xx}} \left( \beta z_o H_z + \varepsilon_{xy} E_y + \varepsilon_{xz} E_z \right) \quad (A.7), \]

where the variable \( \beta = n \sin(\theta) \) is constant from layer to layer by Snell’s law. Now that
the normal components of the electric and magnetic field vectors can be found using the
tangential components the problem proceeds by substituting Eqs. (A.6) and (A.7) into the
remaining four equations found in the sets of equations in (A.4) and (A.5). This
substitution results in the following four equations:

\[
\frac{d}{dx} \begin{pmatrix} E_y \\ H_z \\ E_z \\ -H_y \end{pmatrix} = ik_o \begin{pmatrix} -\beta \frac{\varepsilon_{yy}}{\varepsilon_{xx}} & 1 - \frac{\beta^2}{\varepsilon_{xx}} & -\beta \frac{\varepsilon_{xy}}{\varepsilon_{xx}} & 0 \\ \frac{1}{z_o} \left( \varepsilon_{yy} - \frac{\varepsilon_{yy}^2}{\varepsilon_{xx}} \right) & -\beta \frac{\varepsilon_{yy}}{\varepsilon_{xx}} & 1 \frac{1}{z_o} \left( \frac{\varepsilon_{yy} - \varepsilon_{xy}}{\varepsilon_{xx}} \right) & 0 \\ 0 & 0 & 0 & \frac{1}{z_o} \left( \varepsilon_{yy} - \frac{\varepsilon_{yy}^2}{\varepsilon_{xx}} - \beta^2 \right) \\ \frac{1}{z_o} \left( \varepsilon_{yy} - \frac{\varepsilon_{xy} \varepsilon_{xz}}{\varepsilon_{xx}} \right) & -\beta \frac{\varepsilon_{xy}}{\varepsilon_{xx}} & 1 \frac{1}{z_o} \left( \frac{\varepsilon_{yy} - \varepsilon_{xy}^2}{\varepsilon_{xx}} \right) & 0 \end{pmatrix} \begin{pmatrix} E_y \\ H_z \\ E_z \\ -H_y \end{pmatrix}.
\]

(A.8)

This is rewritten as:

\[
\tilde{\psi}' = \mathbf{\mathcal{V}} \tilde{\psi},
\]

(A.9)

where the vector \( \tilde{\psi} \) is the tangential field vector shown in Eq. (A.8) and the matrix \( \mathbf{\mathcal{V}} \) is
the 4x4 matrix shown in Eq. (A.8) and is defined as the differential propagation matrix.
Eq. (A.9) is the differential equation that must be solved to complete the 4x4 matrix calculation. At this point in the problem the media of interest has not yet been discretized into layers. The differential propagation matrix, before discretization, is a very complicated continuously changing function, with respect to x, and so are the electric and magnetic fields. One way to solve the differential equation in the 4x4 matrix method is to discretize the media into thin layers, this is how the Berreman method is solved. Each layer is assumed to have a constant dielectric permittivity tensor, even with respect to x. The dielectric permittivity matrix of each layer is approximated as the value of that matrix at the center of the layer. Once the media is separated into layers, the differential equation in (A.9) is solved for layer by layer.

The differential equation can be simply solved by diagonalizing the differential propagation matrix. The differential propagation matrix can be diagonalized if there exists a matrix \( \tilde{F} \), such that:

\[
\tilde{\Lambda} = \tilde{F}^{-1} \tilde{\nabla} \tilde{F},
\]

(A.10)

where \( \tilde{\Lambda} \) is a diagonal matrix. For the matrix \( \tilde{\Lambda} \) to be diagonal, the differential propagation matrix, \( \tilde{\nabla} \), must be normal (\( \tilde{\nabla}^H \tilde{\nabla} = \tilde{\nabla} \tilde{\nabla}^H \)) and the matrix \( \tilde{F} \) must be unitary (\( \tilde{F}^H \tilde{F} = \tilde{F} \tilde{F}^H = I \)). The propagation matrix is diagonalized by solving the eigenvalue equation, \( \tilde{\nabla} \tilde{v} = \Lambda \tilde{v} \), where the \( \lambda_i \)'s are the eigenvalues of the propagation matrix and the \( \tilde{v}_i \)'s are the eigenvectors. If \( \tilde{\Lambda} \) is a matrix similar to \( \tilde{\nabla} \) then the eigenvalue equation for \( \tilde{\Lambda} \) is:
\[
\begin{aligned}
\left( \bar{F} \bar{\Lambda} \bar{F}^{-1} \right) \bar{v} &= \lambda \bar{v} \\
\bar{\Lambda} \left( \bar{F}^{-1} \bar{v} \right) &= \lambda \left( \bar{F}^{-1} \bar{v} \right)
\end{aligned}
\]  

(A.11)

where the eigenvectors of $\bar{\Lambda}$ are $\bar{f} = \bar{F}^{-1} \bar{v}$ and the eigenvalues of $\bar{\Lambda}$ are the same as the eigenvalues of the propagation matrix, $\lambda$. The propagation matrix and the matrix $\bar{\Lambda}$ have the same eigenvalues because eigenvalues are preserved in a similarity transform. The eigenvalues of a diagonal matrix are simply its diagonal entries, and the eigenvectors are the columns of diagonalizing matrix $\bar{F}$.

Substituting the relation from Eq. (A.10) into Eq. (A.9), the differential equation becomes $^{43, 44}$:

\[
\bar{\psi}' = \left( \bar{F} \bar{\Lambda} \bar{F}^{-1} \right) \bar{\psi}.
\]  

(A.12)

If we define a new vector $\bar{Y}$, such that $\bar{Y} = \bar{F}^{-1} \bar{\psi}$, the tangential field vector can be written as:

\[
\bar{\psi} = \bar{F} \bar{Y}
\]  

(A.13)

and the derivative of the tangential field vector becomes:

\[
\bar{\psi}' = \bar{F}' \bar{Y} + \bar{F} \bar{Y}'.
\]  

(A.14)

Using Eqs. (A.13) and (A.14) the differential equation is rewritten as:

\[
\bar{Y}' = \left( \bar{\Lambda} - \bar{F}^{-1} \bar{F}' \right) \bar{Y}.
\]  

(A.15)

The derivative of the matrix $\bar{F}$ with respect to $x$ will be zero in this case because the problem is discretized and each layer is assumed to have constant physical properties.
with respect to \( x \); therefore \( \vec{F} \) is independent of \( x \). The elimination of \( \vec{F}' \) results in a diagonalized form of the differential equation:

\[
\vec{Y}' = \Lambda \vec{Y}. \tag{A.16}
\]

Integrating each matrix row separately can easily solve the diagonalized differential equation:

\[
\int_{y_j(0)}^{y_j(d)} \frac{dY_j}{Y_j} = i \kappa \int_0^d \lambda_j dx, \tag{A.17}
\]

where \( d \) is the thickness of the layer being calculated and the subscript "\( j \)" represents each row of the matrix expression in Eq. (A.16), \( j = 1, 2, 3, 4 \).

\[
|Y_j(d)| = e^{i \kappa \lambda_j d} |Y_j(0)| \tag{A.18}
\]

The solution to the diagonalized differential equation simplifies to:

\[
Y_j(d) = e^{i \kappa \lambda_j d} Y_j(0). \tag{A.19}
\]

The eigenvalues are found to be \( \lambda_j = n_j \cos(\theta_j) \), where \( \theta_j \) is the angle of light propagating in the media for eigenvector \( j \), and \( n_j \) is the effective index of refraction seen by eigenvector \( j \). Eq. (A.19) states that the resulting eigenvector component at the end of each layer, \( Y_j(d) \), is simply equal to the eigenvector component at the beginning of the layer, \( Y_j(0) \), times a phase change, \( e^{i \kappa \lambda_j d} \). Thus, while the electric and magnetic fields are propagating through the depth of each layer their orientation stays constant and only the phase changes. The phase matrix used to calculate light propagation through the
depth of the layer is just the exponential of the eigenvalue matrix times the thickness of the layer, $e^{\lambda d}$.

The four eigenvectors in the matrix $\bar{F}$ are the 4 basis fields for the current layer of interest. Two of those vectors are the ordinary and extraordinary vectors propagating forward and the other two are the ordinary and extraordinary vectors propagating backward. The total tangential field components are conserved across boundaries; therefore, those field components are propagated from one layer to the next and are then transformed into the field components in the layer. Once in the layer, the phase change accumulated due to propagating through that layer is calculated using the $\bar{F}$ matrix and the eigenvectors:

$$\tilde{\psi}_d = \bar{F}\tilde{\psi}_d = \bar{F}e^{\lambda d}\tilde{\psi}_0 = \bar{F}e^{\lambda d}\bar{F}^{-1}\tilde{\psi}_0. \quad (A.20)$$

By repeating this process many layers can be accounted for. The result is a simple matrix multiplication for each layer:

$$\tilde{\psi}_{\text{exit}} = \left(\bar{F}_N e^{\lambda N d}\bar{F}_N^{-1}\right)_{\text{layer } N} \cdots \left(\bar{F}_2 e^{\lambda 2d}\bar{F}_2^{-1}\right)_{\text{layer } 2} \left(\bar{F}_1 e^{\lambda 1d}\bar{F}_1^{-1}\right)_{\text{layer } 1} \tilde{\psi}_{\text{enter}} \quad (A.21)$$

where there are $N$ layers in the stack and each layer has the thickness $d$. We choose isotropic first and last layers so that the magnetic field can be easily related to the electric field using constants of the equation.

The Berreman method assumes a completely coherent light source, which is generally not the case; therefore, a method must be devised to average out the interference between layers that are farther apart then the coherence length of the light source being modeled. The interference “ringing” in the calculated data is the same type
of “ringing” which is seen in etalons. A documented method for eliminating the “ringing” is inserting an apodization layer into the stack between two layers which have large differences in index \(^45\). Unfortunately, this method does not allow one to control the partial coherence of the source.

An alternative method to eliminate the “ringing” can be arrived at by looking at two aspects of partially coherent sources. First, consider that the coherence length of the light source is directly related to the linewidth of the light source. The linewidth is simulated by averaging together the reflectance or transmittance at several wavelengths. For example, if the linewidth of the monochrometer used to measure the spectra is 2 nm, then the calculated reflectance is averaged over a 2 nm distribution through out the entire spectra. The larger the linewidth the less “ringing” in the spectra; therefore, one part of modeling an incoherent source is wavelength averaging the data. The degree of coherence can then be controlled by the wavelength range over which averaging is done. The longitudinal coherence length is related to the linewidth by the following relation:

\[
\Delta l = c \Delta t \sim \frac{c}{\Delta \nu} \frac{\lambda^2}{\Delta \lambda_0},
\]  

where \(\Delta l\) is the longitudinal coherence length, \(c\) is the speed of light, \(\Delta t\) is the coherence time, \(\Delta \nu\) is the frequency bandwidth, \(\bar{\lambda}\) is the mean wavelength, and \(\Delta \lambda_0\) is the linewidth of the light beam \(^5\). Light from two points more than a coherence length apart is, to a good approximation, incoherent.

The other issue that must be considered is pointed out in the third of the Fresnel-Argo Laws; for a coherent light source, two orthogonal polarization states are coherent;
and for an incoherent light source, two orthogonal polarization states are incoherent. Therefore, if the light source is incoherent and s polarized light (perpendicular to the plane of incidence) is rotated onto the p polarization axis (parallel to the plane of incidence), those two wave fronts can not interfere with one another due to the fact that they are incoherent. This can be reflected in the Berreman calculation by paying close attention to the cross terms between the s polarized state and the p polarized state. For light transmitted through one stack the output s and p components of the electric field have the form:

$$E_{s}^{\text{out}} = A_{s}E_{s}^{\text{in}} + B_{s}E_{p}^{\text{in}},$$
$$E_{p}^{\text{out}} = A_{p}E_{s}^{\text{in}} + B_{p}E_{p}^{\text{in}},$$

(A.23)

where A and B are determined by the problem. For coherent light, the transmitted energy is:

$$T_{\text{coherent}} = \frac{\left( |E_{s}^{\text{out}}|^{2} + |E_{p}^{\text{out}}|^{2} \right)}{\left( |E_{s}^{\text{in}}|^{2} + |E_{p}^{\text{in}}|^{2} \right)}.$$  (A.24)

For incoherent light incident on one stack, the transmitted energy is:

$$T_{\text{incoherent}} = \frac{\left( |A_{s}E_{s}^{\text{in}}|^{2} + |B_{s}E_{p}^{\text{in}}|^{2} + |A_{p}E_{s}^{\text{in}}|^{2} + |B_{p}E_{p}^{\text{in}}|^{2} \right)}{\left( |E_{s}^{\text{in}}|^{2} + |E_{p}^{\text{in}}|^{2} \right)}.$$  (A.25)

This eliminates any cross terms between the s and p components that occur in Eq. (A.24) and are not allowed for incoherent light. For light transmitted through two stacks the output s and p components of the electric field have the form:
\[ E_{s}^{out2} = C_s E_{s}^{out1} + D_s E_{p}^{out1} = C_s \left( A_s E_{s}^{in} + B_s E_{p}^{in} \right) + D_s \left( A_p E_{s}^{in} + B_p E_{p}^{in} \right) \]
\[ E_{p}^{out2} = C_p E_{s}^{out1} + D_p E_{p}^{out1} = C_p \left( A_s E_{s}^{in} + B_s E_{p}^{in} \right) + D_p \left( A_p E_{s}^{in} + B_p E_{p}^{in} \right) \]

where \( E_{s}^{out1} \) and \( E_{p}^{out1} \) are simply numbers that can not be separated into s and p components. The values \( E_{s}^{out1} \) and \( E_{p}^{out1} \) are expressed in s and p components in Eq. (A.26) simply to demonstrate that each value depends on both the input s and p components and that the s and p polarization dependence can’t be eliminated by just removing the cross terms as is done above. Because the values \( E_{s}^{out1} \) and \( E_{p}^{out1} \) can’t be separated into s and p components the method of writing the transmittance for incoherent light, in Eq. (A.25), can’t be used here; therefore, a new method needs to be devised for incoherent light incident on multiple stacks. The solution is to input s polarized light \((0,1)\) into the problem and propagate the calculation through all stacks, with cross terms included, Eq. (A.3) is used to calculated the output intensity for s polarized light. Then input p polarized light \((1,0)\) into the problem, and propagate the calculation through all stacks resulting in a second set of results, either transmittance or reflectance for p polarized light. The final transmittance or reflectance is found by averaging together the s polarized results and the p polarized results for the entire spectra 39. There is a very noticeable difference between the incoherent and coherent calculation when the incident angle is increased.
A.2 How to Transform Between Multiple Stacks

A.2.1 Why a Transformation between Multiple Stacks is Necessary

The transformations that allow light to propagate through successive Berreman stacks, which have non-parallel axis of dielectric variation, are developed here. In general, two transformation processes need to be defined; one for the case where light is transmitted through one stack and is then incident on a second stack; and the other is for the case where light is reflected from one stack and then back propagated through either a third stack or the initial stack. The case where light is reflected from one stack and then back propagated through the initial stack is found by extending the case in which light is back propagated through a third stack. Here, we will discuss those transformations; first, the case where light is transmitted from one stack and is then incident on a second stack, and second, the case where light is reflected from the second stack and back propagated through a third stack.

Calculating light propagation requires two sets of parameters; those which define the electric field vector: wavelength, angle of incidence, and polarization state, and those which define the material: indices of refraction, optic axis orientation, and layer thickness. If the complete optical system contains multiple optical stacks that have non-parallel axes of dielectric variation, a coordinate system must be defined for each optical stack, such that the coordinate system’s x-axis is defined as the direction of dielectric variation. The axes of dielectric variation can also be referred to as the layer normal since the discretized layers are defined to be perpendicular to the axes of dielectric variation. A coordinate system is defined for each optical stack. The material parameters
of each stack are defined in its respective coordinate system. The electric field vector parameters are defined in the coordinate system in which light is currently propagating through; therefore, a set of electric field vector parameters are defined for each coordinate system.

The first stack is birefringent stack one, or B1 stack. The second stack is birefringent stack 2, or B2 stack. The final stack is birefringent stack three, or B3 stack. A coordinate system must be defined for each optical stack. The lab fixed frame, which is parallel to the B1 frame, (xyz) is defined as having the x-axis along the layer normal direction of the B1 stack. The Berreman B1 frame (x'y'z') rotates with the wave vector and is defined as having the x'-axis parallel to the x-axis and the incident wave vector always lies in the x'y' plane, this is the axes frame which is previously described with the Berreman method in A.1.2. The coordinates describing the optic axes in each layer of the B1 frame are gotten from their definitions in the B1 frame by a rotation about the x-axis by an angle ϕ. The x'y'z' axes are easily found from the lab frame by rotating all axes along the principal axis of the stack. In the B1 stack, the rotated material fixed frame happens to coincide with the lab fixed frame. We assume that the second stack is tilted with regard to the first stack, as shown in Fig. A-3. The tilted material fixed frame, or B2 frame, (x''y''z'') has the x''-axis along the layer normal direction of the B2 stack. The z''-axis is defined as the rotation axis of the B2 stack relative to the lab fixed frame (Fig. A-3). The B2 tilted material fixed frame defines a tilting of the layer discretization and the normal to the dielectric variation meaning the x''-axis occurs at some angle to the first stack's x-axis. The Berreman B2 frame (x'''y'''z''') rotates with the wave vector around
the \( x''''' \)-axis, which is defined as the axis of rotation for the Berreman B2 frame. The \( x'''''y'''''z''''' \) system has the \( x''''' \)-axis parallel to the \( x'' \)-axis and is defined so that the incident wave vector is always in the \( x'''''y''''' \) plane. In the B3 stack, the rotated material fixed frame, or B3 frame, \( (x^Ry^Rz^R) \) has the \( x^R \)-axis along the layer normal direction of the B3 stack and positive \( x^R \) is in the direction of the wave vector; therefore, the B3 frame is rotated 180° around the \( y^R \)-axis (Fig. A-4). The Berreman B3 frame \( (x^Ry^Rz^R) \) rotates with the wave vector around the \( x^R \)-axis, which is defined as the axis of rotation for the Berreman B3 frame. The rotating coordinate systems are mentioned here because those are the coordinate systems used to input variable parameters into the Berreman method. From here on we will discuss transformations from only the material fixed axes since the transformation to the rotating axes is just a redefinition of the azimuthal angle of the optic axes in the birefringent layers.

At each coordinate transformation the light propagation calculation is suspended and the electric field is calculated, this has the effect of eliminating any coherent interference between stacks. We split the optical system up in this fashion to calculate the coordinate system transformations required to complete the Berreman calculation in each stack. An added benefit, is that some of the etalon effects between the stacks are eliminated. As stated earlier, the Berreman method calculates two sets of tangential electric field components, two forward propagating and two reverse propagating. The forward propagating, transmitted components exiting the B1 stack (the result of the Berreman calculation) are used as input forward propagating components to the B2 stack. The reverse propagating reflected components from the B2 stack (the result of the
Berreman calculation) are used as the input forward propagating components to the B3 stack. In each calculation one set of tangential components are discarded, thus preventing any interference between the forward and reflected modes from the different stacks (see Fig. A-5). This is desirable because it is unlikely that the stacks are separated by a distance smaller than the coherence length of the illuminating light source.
FIG. A-3. Relation between the B1 frame and the B2 frame, where, \(|\vec{k}| = |\vec{k''}|\) and \(\vec{k} \parallel \vec{k''}\).

The B1 axes are shown in (a) and the B2 axes are shown in (b).
FIG. A-4. Relation between the B3 frame and the B2 frame, where, $|\vec{k}| = |\vec{k}''|$ and $\vec{k} \parallel \vec{k}''$. The B2 axes are shown in (a) and the B3 axes are shown in (b).
FIG. A-5. Flow chart illustrating which electric field components and axes are used for each stack.
A.2.2 Forward Propagating Transformation

Once the required axes are identified the material variables can be defined in their respective axes frames and the electric field vector transformations from one stack to the next are found. The incident electric field vector is initially described in the B1 frame along with the B1 stack’s material properties, while the B2 stack material properties are described in the B2 frame. After the electric field vector is propagated through the B1 stack, using the Berreman 4x4 method, it is transformed, using rotation matrices, into the B2 frame. Once the electric field vector is described in the new axes frame it is propagated through the B2 stack using the Berreman 4x4 method. The electric field vector reflected from the B2 stack is transformed into the reflected B1 frame and then propagated through the reverse B1 stack, using the Berreman 4x4 method, resulting in the complete system’s reflected electric field vector.

Light is propagated through the B1 stack using the Berreman method, the electric field vector and the optic axes of the B1 stack components are defined in the B1 Berreman axes for this calculation. The following considers how to rotate the output electric field vectors from the B1 stack material axes into the B2 stack material axes. The transformation from the Berreman axes to the material axes and vice versa are completed before the light is propagated through the B2 stack using the Berreman method. To begin the transforming of the output electric field vector from the B1 stack material axes to the B2 stack material axes the wave vector, $\vec{k}$, is defined in the last layer of the B1 frame:

$$
\begin{bmatrix}
k_x \\
k_y \\
k_z
\end{bmatrix} =
\begin{bmatrix}
k \cos(\theta) \\
k \sin(\theta) \cos(\varphi) \\
k \sin(\theta) \sin(\varphi)
\end{bmatrix}
$$

(A.27)
where the first layer is described as an isotropic layer (for simplicity), $k = 2\pi/\lambda$ ($\lambda$ is the wavelength of light in the isotropic layer), $\theta$ is the polar angle, and $\phi$ is the azimuthal angle. Both angles are defined in Figs. A-1, A-2, and A-4. The total field components, $E_p$ and $E_\phi$, are related to the tangential field components that are input into the Berreman calculation in the following way:

$$
\begin{align*}
E_y &= E_p \cos(\theta) \\
E_z &= E_\phi
\end{align*}
$$

(A.28)

Once the Berreman calculation is completed for the B1 stack, the exiting tangential electric field components are transformed back into the total field components using the relation above and the total field is then transformed into the B2 frame. Transformation from the B1 frame to the B2 frame requires a tilt along the $z$-axis, by an angle $\theta$, using the following rotation matrix:

$$
R_z(\theta) = \begin{pmatrix}
\cos(\theta) & \sin(\theta) & 0 \\
-\sin(\theta) & \cos(\theta) & 0 \\
0 & 0 & 1
\end{pmatrix}
$$

(A.29)

Before transforming the electric field vectors that exited the B1 stack in the B2 frame, the wave vector and the angles defining it must first be transformed into the B2 frame. The transformation for the wave vector is defined using the rotation matrix in Eq. (A.29):

$$
\begin{pmatrix}
k''_x \\
k''_y \\
k''_z
\end{pmatrix} =
\begin{pmatrix}
\cos(\theta) & \sin(\theta) & 0 \\
-\sin(\theta) & \cos(\theta) & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
k_x \\
k_y \\
k_z
\end{pmatrix}
= \begin{pmatrix}
k_x \cos(\theta) + k_y \sin(\theta) \\
-k_x \sin(\theta) + k_y \cos(\theta) \\
k_z
\end{pmatrix}
$$

(A.30)

The wave vector in the B2 frame is:
\[ k''_{x'} = k(\cos(\theta) \cos(h) + \sin(\theta) \cos(\varphi) \sin(h)) \]
\[ k''_{y'} = k(-\cos(\theta) \sin(h) + \sin(\theta) \cos(\varphi) \cos(h)) \]  
\[ k''_{z'} = k \sin(\theta) \sin(\varphi) \]  

(A.31)

Using the wave vector in the B2 frame the polar and azimuthal angles are defined in the same frame:

\[ \theta'' = \cos^{-1}\left(\frac{k''_{x'}}{k_o}\right) = \cos^{-1}\left(\cos(\theta) \cos(h) + \sin(\theta) \cos(\varphi) \sin(h)\right) \]
\[ \varphi'' = \tan^{-1}\left(\frac{k''_{x'}}{k''_{y'}}\right) = \tan^{-1}\left(\frac{\sin(\theta) \sin(\varphi)}{\cos(h) + \sin(\theta) \cos(\varphi) \cos(h)}\right) \]  

(A.32)

For the simplified case where the plane of incidence for both the B1 and B2 coordinate systems are the same (\(\varphi = 0\)), the polar angle in the B2 frame is found to be: \(\theta'' = (\theta - h)\), which can be geometrically derived in Fig. A-3, and the azimuthal angle is found to be: \(\varphi'' = \tan^{-1}\left(\frac{0}{\sin(\theta - h)}\right)\), which makes \(\varphi'' = 0\) (the arctangent term is equal to 0\(^{\circ}\), because the denominator is positive). The polar angle, \(\theta''\), defines the Snell's law quantity, \(\beta'' = n_{\text{air}} \sin(\theta'')\), that is used in the Berreman method for propagation from one layer to the next and is the same for all plane waves traveling in one stack. The azimuthal angle, \(\varphi''\), is used to define the B2 frame.

The inputs and outputs of the Berreman calculations are the tangential components of the electric field vector, \(E_y\) and \(E_z\). The tangential field components are found from the total field components, which are typically defined along the s and p polarization axes. The s and p polarization axes are defined such that the s polarization axis is perpendicular to the plane of incidence and the p polarization axis is in the plane...
of incidence and perpendicular to the wave vector and the \( s \) polarization axis. In general, the plane of incidence for the B1 stack and the plane of incidence of the B2 stack are not parallel, as shown in Fig. A-6. When the incident light is in the \( xy \) plane (azimuthal angle, \( \phi \), equals zero), all three layers have the same plane of incidence. This is the only azimuthal angle in which this situation occurs. Since these planes are not parallel in general, the \( s \)-polarization axis will not be the same for the two axes frames; therefore, a transformation of the total electric field vector must be established. This transformation is a rotation of the total field components, \( E_s \) and \( E_p \), about the wave vector, \( \vec{k}'' \), where the rotation angle is the angle between the unit vector perpendicular to the plane of incidence in the B1 stack, \( \hat{S}_{xy} \) (the subscript denotes that the vector components are defined in the B1 frame), and the unit vector perpendicular to the plane of incidence in the B2 stack, \( \hat{S}''_{x'y''z''} \) (see Fig. A-7). Our approach to finding the rotation angle will be to first define these two unit vectors, and then to express both of them in the \( x''y''z'' \) frame to find their included angle.

The vector \( \hat{S}_{xy} \) is defined using the cross product of \( \hat{k} \) and \( \hat{x} \).

\[
\hat{S}_{xy} \parallel \hat{x} \times \hat{k} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 0 & 0 \\ \frac{k_x}{k} & \frac{k_y}{k} & \frac{k_z}{k} \end{vmatrix} = \begin{pmatrix} 0 \\ -\frac{k_x}{k} \\ \frac{k_y}{k} \end{pmatrix} \tag{A.33}
\]

and

\[
|\hat{x} \times \hat{k}| = \sin(\theta). \tag{A.34}
\]
Solving for $\hat{S}_{xyz}$,

$$\hat{S}_{xyz} = \frac{\hat{x} \times \hat{k}}{\sin(\theta)} = \begin{pmatrix} 0 \\ -\sin(\varphi) \\ \cos(\varphi) \end{pmatrix}. \quad (A.35)$$

In the B1 frame, $\hat{S}_{xyz}$ is not a function of $x$ (or $\theta$) by definition. Notice that $\hat{S}_{xyz}$ is perpendicular to the projection of $\hat{k}$ in the $yz$ plane, as it should be by definition. To rotate $\hat{S}_{xyz}$ into the B2 frame ($x''y''z''$) use the rotation matrix defined in Eq. (A.29).

$$\hat{S}_{x'y'z''} = \begin{pmatrix} \cos(h) & \sin(h) & 0 \\ -\sin(h) & \cos(h) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{S}_x \\ \hat{S}_y \\ \hat{S}_z \end{pmatrix} \quad (A.36)$$

Multiply this out using Eq. (A.35):

$$\hat{S}_{x'} = -\sin(\varphi)\sin(h)$$

$$\hat{S}_{y'} = -\sin(\varphi)\cos(h). \quad (A.37)$$

$$\hat{S}_{z'} = \cos(\varphi)$$

Due to the fact that the B2 stack has a different plane of incidence a new unit vector $\hat{S}''_{x'y'z''}$ is defined in the B2 stack. Using the wave vector in Eq. (A.31):

$$\hat{S}''_{x'y'z''} \parallel \hat{x}'' \times \hat{k}'' = \begin{vmatrix} \hat{x}'' & \hat{y}'' & \hat{z}'' \\ 1 & 0 & 0 \\ \frac{k''_x}{k} & \frac{k''_y}{k} & \frac{k''_z}{k} \end{vmatrix} = \begin{pmatrix} 0 \\ \frac{k''_z}{k} \\ \frac{k''_y}{k} \end{pmatrix} \quad (A.38)$$
Solving for $\hat{S}_{x'y'z'}^n$:

$$
\hat{S}_{x'y'z'}^n = \frac{\mathbf{x} \times \mathbf{k}^n}{\sin(\theta'')}
\begin{pmatrix}
0 \\
-\sin(\theta)\sin(\varphi) \\
\sin(\theta'') \\
-\cos(\theta)\sin(h) + \sin(\theta)\cos(\varphi)\cos(h) \\
\sin(\theta'')
\end{pmatrix}
$$

(A.39)

The angle between $\hat{S}_{x'y'z'}^n$ and $\hat{S}_{x'y'z'}^n$ is defined via the dot product:

$$
\hat{S}_{x'y'z'}^n \cdot \hat{S}_{x'y'z'}^n = \cos(\rho),
$$

(A.40)

where $\rho$ is the angle between $\hat{S}_{x'y'z'}^n$ and $\hat{S}_{x'y'z'}^n$ in the $x''y''z''$ axes, Fig. A-8.

$$
\rho = \cos^{-1}\left(\hat{S}_{x'y'z'}^n \cdot \hat{S}_{x'y'z'}^n\right) = \cos^{-1}\left(\frac{\sin(\theta)\cos(h) - \cos(\theta)\cos(\varphi)\sin(h)}{\sin(\theta'')}\right)
$$

(A.41)

Now that the angle between $\hat{S}_{x'y'z'}^n$ and $\hat{S}_{x'y'z'}^n$ is defined, it can be used to rotate $E_s$ and $E_p$ to the proper locations such that $E_s''$ fulfills the definition which states that it is perpendicular to the plane of incidence in the B2 stack. To rotate $E_p$ and $E_s$ around $\mathbf{k}^n$ and align $E_s''$ with $\hat{S}_{x'y'z'}^n$, a two-dimensional rotation matrix is used:

$$
\begin{pmatrix}
E_p^n \\
E_s^n
\end{pmatrix} =
\begin{pmatrix}
\cos(\rho) & \sin(\rho) \\
-\sin(\rho) & \cos(\rho)
\end{pmatrix}
\begin{pmatrix}
E_p \\
E_s
\end{pmatrix}
$$

(A.42)

$$
E_p'' = E_p\cos(\rho) + E_s\sin(\rho)
$$

$$
E_s'' = -E_p\sin(\rho) + E_s\cos(\rho)
$$

(A.43)

For the case where the B1 stack and B2 stack have the same plane of incidence ($\varphi = 0$) the angle $\rho$ becomes:
\[
\rho_{\varphi=0} = \cos^{-1}\left( \frac{\sin(\theta) \cos(h) - \cos(\theta) \sin(h)}{\sin(\vartheta'')} \right) = \cos^{-1}(1) = 0 , \quad (A.44)
\]

where

\[
\sin(\theta'')_{\varphi=0} = \sin(\theta - h) . \quad (A.45)
\]

This means that the total field in the B1 frame is equal to the total field in the B2 frame when the azimuthal angle, \( \varphi \), equals zero:

\[
\begin{align*}
E_p''_{\varphi=0} &= E_p \\
E_s''_{\varphi=0} &= E_s .
\end{align*} \quad (A.46)
\]

This result is expected for the case where the two stacks have the same plane of incidence. The rotated \( E_p'' \) and \( E_s'' \) are transformed into the tangential components and propagated through the B2 stack using the Berreman calculation. Once the Berreman calculation is completed for the B2 stack, the exiting, reflected tangential electric field components are transformed back into the total field components using a relation similar to the one in Eq. (A.28) and the total field components are then transformed into the reverse B1 frame.
FIG. A-6. When $\vec{k}$ is in the xz frame the $E_s$ component is along the y axis. The figure above shows that $E_s$ is not parallel to $E_s''$. 
FIG. A-7. The total field components, $E_s$ and $E_p$, are transformed from the B1 stack to the B2 stack by rotating around the wave vector an angle $\rho$, where $E_s \parallel S_{xyz}$ and 

$$E''_s \parallel S''_{xyz}. $$
A.2.3 Reverse Propagating Transformation

The final Berreman calculation simulates light propagation through the third optical stack, the B3 stack. The Berreman discretization described earlier is completed separately for each stack, with no knowledge that previous or further calculations are being done; therefore, when considering how the axes are oriented in each stack they are simply defined with respect to the wave vector. The general axes definition for the Berreman method is shown in Fig. A-2. When considering the reflected axes refer to Fig. A-8, which is simply Fig. A-2 inverted. The Berreman axes are defined as a right handed coordinate system in which light propagates through the first quadrant, +x-axis and +y-axis. Transformation from the B2 frame to the B3 frame requires the axes to be rotated into the B3 frame using an inverse rotation similar to the steps taken in Eqs. (A.30) through (A.43). The inverse rotation is not a reversal of the rotation described above because the x-axis and the x''-axis are not parallel to one another.

The reverse propagation transformation begins with a similar approach as that taken by the forward propagation transformation. Before the electric field vector reflected from the B2 stack is defined in the B3 frame, the wave vector and its angles are defined. At this point, the wave vector has been reflected and now has a negative x component,

$$\vec{k}^{uR} = \begin{pmatrix} -k''_x \\ k''_y \\ k''_z \end{pmatrix}. \quad (A.47)$$

The first transformation accounts for the axes redefinition:
\[
\begin{pmatrix}
  k_x^R \\
  k_y^R \\
  k_z^R
\end{pmatrix}
= \begin{pmatrix}
  -1 & 0 & 0 \\
  0 & 1 & 0 \\
  0 & 0 & -1
\end{pmatrix}
\begin{pmatrix}
  -k_x^R \\
  k_y^R \\
  -k_z^R
\end{pmatrix}
= \begin{pmatrix}
  k_x \\
  k_y \\
  -k_z
\end{pmatrix},
\]

(A.48)

this will be referred to as the reflected wave vector in the reflected B2 frame. The polar and azimuthal angles will be defined later and this rotation will be apparent in those definitions. The second transformation completed describes the wave vector in the B3 frame. This transformation is similar to the one completed in Eq. (A.30):

\[
\begin{pmatrix}
  k_{r_x}^R \\
  k_{r_y}^R \\
  k_{r_z}^R
\end{pmatrix}
= \begin{pmatrix}
  \cos(h) & \sin(h) & 0 \\
  -\sin(h) & \cos(h) & 0 \\
  0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  k_{x'}^R \\
  k_{y'}^R \\
  -k_{z'}^R
\end{pmatrix}
= \begin{pmatrix}
  k_{r_x} \cos(h) + k_{y'} \sin(h) \\
  -k_{z'} \sin(h) + k_{y'} \cos(h) \\
  -k_{z'}
\end{pmatrix}.
\]

(A.49)

The rotation matrix used in Eq. (A.49) is the same as the rotation matrix defined in Eq. (A.29), this is because the x-axis and z-axis are now pointing in the opposite direction.

The wave vector defined in the B3 frame is:

\[
k_{x'}^R = k \left( \cos(\theta) \cos(2h) + \sin(\theta) \cos(\varphi) \sin(2h) \right)
\]

\[
k_{y'}^R = k \left( -\cos(\theta) \sin(2h) + \sin(\theta) \cos(\varphi) \cos(2h) \right).
\]

(A.50)

\[
k_{z'}^R = -k \sin(\theta) \sin(\varphi)
\]

Notice that Eq. (A.50) is similar to Eq. (A.31), the difference is the angle (2h) instead of (h). Using the reflected wave vector the polar and azimuthal angles are described in the reverse direction:

\[
\theta^R = \cos^{-1}\left( \frac{k_x^R}{k_o} \right) = \cos^{-1}\left( \cos(\theta) \cos(2h) + \sin(\theta) \cos(\varphi) \sin(2h) \right)
\]

\[
\varphi^R = \tan^{-1}\left( \frac{k_z^R}{k_y^R} \right) = \tan^{-1}\left( \frac{-\sin(\theta) \sin(\varphi)}{-\cos(\theta) \sin(2h) + \sin(\theta) \cos(\varphi) \cos(2h)} \right).
\]

(A.51)
For the simplified case where there is only one plane of incidence \((\varphi = 0)\) for all three stacks, the polar angle is found to be: \(\theta^R = (\theta - 2h)\), which can be geometrically derived in Fig. A-4, and the azimuthal angle is found to be: \(\varphi^R = \tan^{-1}\left(0/\sin(\theta - 2h)\right)\), which makes \(\varphi^R = 0\) (the arctangent term is equal to 0°, because the denominator is positive).

Similar to the derivation above, the electric field components must be rotated around the reflected wave vector to transform them into the B3 frame. The angle of rotation of the electric field components is found using the unit vector perpendicular to the reflected plane of incidence in the B2 frame, \(\mathbf{S}^{nR}_{x'y'z'^*}\), and the unit vector perpendicular to the backwards propagating plane of incidence in the B3 frame, \(\mathbf{S}^{R}_{x'y'z'}\).

The unit vector perpendicular to the reflected plane of incidence in the B2 frame, \(\mathbf{S}^{nR}_{x'y'z'^*}\), is pointing the opposite direction of \(\mathbf{S}^{nR}_{x'y'z'^*}\) because it is found using the cross product of the reflected wave vector in the B2 frame and the \(x'^*\)-axis, this direction change accounts for the \(\pi\) phase shift upon reflection from a denser medium:

\[
\mathbf{S}^{nR}_{x'y'z'^*} \parallel \mathbf{\tilde{x}}^n \times \mathbf{\tilde{k}}^{nR} = \begin{vmatrix}
\hat{x}^n & \hat{y}^n & \hat{z}^n \\
1 & 0 & 0 \\
\frac{k''}{k} & \frac{k''}{k} & -\frac{k''}{k}
\end{vmatrix} \left(\begin{array}{c}
0 \\
\frac{k''}{k} \\
\frac{k''}{k}
\end{array}\right) \quad (A.52)
\]

Both unit vectors, \(\mathbf{S}^{nR}_{x'y'z'^*}\) and \(\mathbf{S}^{R}_{x'y'z'}\), must be defined in the B3 frame to find the angle between them. The unit vector perpendicular to the reflected plane of incidence in the B2 frame, \(\mathbf{S}^{nR}_{x'y'z'^*}\), is rotated into the \(x^R y^R z^R\) axes:
\[
\hat{S}^n_{x,y,z} = \begin{pmatrix}
\cos(h) & \sin(h) & 0 \\
-k_y & k_y & 0 \\
0 & 0 & 1
\end{pmatrix},
\]
where,
\[
\hat{S}^n_x = \frac{\sin(\theta)\sin(\phi)\sin(h)}{\sin(\theta^n)} \\
\hat{S}^n_y = \frac{\sin(\theta)\sin(\phi)\cos(h)}{\sin(\theta^n)} \\
\hat{S}^n_z = \frac{-\cos(\theta)\sin(h) + \sin(\theta)\cos(\phi)\cos(h)}{\sin(\theta^n)}
\]

The unit vector perpendicular to the back propagating plane of incidence in the B3 frame, \(\hat{S}^R_{x,y,z}\), is defined using the cross product of the reflected unit wave vector and \(\hat{x}\):
\[
\hat{S}^R_{x,y,z} \parallel \hat{x} \times \hat{k}^R = \begin{pmatrix}
\hat{x} & \hat{y} & \hat{z} \\
1 & 0 & 0 \\
\frac{k_x}{k} & \frac{k_y}{k} & \frac{k_z}{k}
\end{pmatrix} \begin{pmatrix}
0 \\
-k_x^R \\
-k_y^R \\
-k_z^R \\
\end{pmatrix}.
\]

Using the wave vector in Eq. (A.50) the unit vector perpendicular to the back propagating plane of incidence in the B3 frame is described as:
\[
\hat{S}^R_{x'y'z'} = \frac{\hat{x} \times \hat{k}^R}{\sin(\theta^R)} = \begin{pmatrix}
0 \\
\frac{\sin(\theta)\sin(\varphi)}{\sin(\theta^R)} \\
-\cos(\theta)\sin(2h) + \sin(\theta)\cos(\varphi)\cos(2h)
\end{pmatrix}.
\] (A.56)

The angle between the two unit vectors is defined using the dot product:
\[
\hat{S}^R_{x'y'z'} \cdot \hat{S}^n_{x'y'z'} = \cos(\rho^R).
\] (A.57)

Solving for the angle \(\rho^R\):
\[
\rho^R = \cos^{-1}\left(\hat{S}^R_{x'y'z'} \cdot \hat{S}^n_{x'y'z'}\right) = \cos^{-1}\left(\frac{\sin^2(\theta)\sin^2(\varphi)\cos(h)}{\sin(\theta^R)\sin(\theta^n)}\right) + \frac{\cos(\theta)\sin(2h) - \sin(\theta)\cos(\varphi)\cos(2h)}{\sin(\theta^R)\sin(\theta^n)}. \tag{A.58}
\]

Now that the angle between \(\hat{S}^n_{x'y'z'}\) and \(\hat{S}^R_{x'y'z'}\) is defined, it can be used to rotate \(E_p''\) and \(E_s''\) to the proper locations such that \(E_{nR}\) fulfills the definition which states that it is perpendicular to the back propagating plane of incidence in the B3 stack. The rotation of \(E_p''\) and \(E_s''\) around \(\hat{k}^R\) results in:
\[
\begin{pmatrix}
E_p^R \\
E_s^R
\end{pmatrix} = \begin{pmatrix}
\cos(\rho^R) & \sin(\rho^R) \\
-\sin(\rho^R) & \cos(\rho^R)
\end{pmatrix}\begin{pmatrix}
E_p'' \\
E_s''
\end{pmatrix}
\]
\[
E_p^R = E_p''\cos(\rho^R) + E_s''\sin(\rho^R) \tag{A.59}
\]
\[
E_s^R = -E_p''\sin(\rho^R) + E_s''\cos(\rho^R)
\]
For the simplified case where the B2 stack and the B3 stack have the same plane of incidence \((\varphi = 0)\) the angle \(\rho^R\) becomes:

\[
\rho^R_{\varphi=0} = \cos^{-1} \left( \frac{\sin(\theta - 2h) \cdot \sin(\theta - h)}{\sin(\theta^R) \cdot \sin(\theta^\nu)} \right) = \cos^{-1} (1) = 0, \quad (A.60)
\]

where

\[
\sin(\theta^R)_{\varphi=0} = \sin(\theta - 2h). \quad (A.61)
\]

This results in the total field in the B2 frame equal to the total field in the B3 frame for the case where the two stacks have the same plane of incidence:

\[
\begin{align*}
E^R_{p\varphi=0} &= E^\nu_p \\
E^R_{s\varphi=0} &= E^\nu_s 
\end{align*} \quad (A.62)
\]

Once \(E^R_p\) and \(E^R_s\) are found they are related to the tangential field components by an equation similar to that in Eq. (A.28) and can be used to for the Berreman calculation in the B3 stack. After the reflected wave vector and the electric field vector are rotated into the B3 stack they are propagated through that stack using the Berreman 4x4 method.
FIG. A-8. Reverse axes defined in terms of the wave vector in the same manner that the axes are defined in figure A-2.
A.2.4 Modeling a non-tilted stack of tilted layers

When the B2 layer is tilted the path length either increases or decreases due to the change in angle between the layer normal and the wave vector, see Fig. A-9. The thickness of the tilted layer needs to be adjusted so that the retardation through the tilted layer is equal to the retardation through the untilted layer. For the case where the tilted layer is isotropic, the thickness of the tilted layer is found using the path length of the untilted layer,

\[ d_{\text{new}} = d \frac{\cos \left[ \frac{n}{n_1} \sin \left( \theta^* \right) \right]}{\cos \left[ \frac{n}{n_1} \sin \left( \theta \right) \right]} \]  \hspace{1cm} (A.63)

Where \( d_{\text{new}} \) is the thickness of the tilted layer, \( d \) is the thickness of the untilted layer, \( n \) is the index of the surrounding material, \( n_1 \) is the index of the tilted material, and the angles are defined in Fig. A-9. Notice that the angle \( \theta^* \) is a function of the incident polar, \( \theta \), and azimuthal, \( \varphi \), angles along with the slant angle, \( h \); therefore the value of \( d_{\text{new}} \) is also a function of those angles.

For the case where the tilted layer is birefringent, the thickness of that layer is found using its birefringence instead of the angular method, previously described. This is because the angular method does not account for the path length of both the ordinary and the extraordinary rays. To find the birefringent tilted layer's thickness, the retardation \( (\Delta n_{\text{untilted}} d) \) of the untilted layer and the birefringence \( (\Delta n_{\text{tilted}}) \) of tilted layer are first calculated at each incident polar, \( \theta \), and azimuthal angle, \( \varphi \), for the wave vector. Then, the thickness of the tilted layer is calculated using those values:
\[ d_{new} = \frac{\Delta n_{untiled} d}{\Delta n_{tilted}} \] (A.64)

Once again, the tilted layer’s thickness is a function of the incident polar, \( \theta \), and azimuthal, \( \varphi \), angles along with the slant angle, \( h \).
Fig. A-9. (a) An untitled B2 stack, \(d\) is the layer thickness and \(L\) is the path length in that layer. (b) A tilted B2 stack, the path length in the tilted layer, \(L\), must be equal to the path length in the untitled layer, \(L\), so that equal retardation results from the two separate cases. The value of \(d_{\text{new}}\) is found from \(L\).
A.2.5 Special Case: Final Stack Equals Initial Stack for Back Propagating Waves

This above described method is demonstrated with an optical system containing three separate stacks. In this example, we will consider three optical stacks the first consisting of a polarizer, retarder, and liquid crystal, the second consisting of a holographic reflector, and the third is the first stack in reflection. This method is not limited to this particular optical system. All stacks are birefringent layers that are discretized into many thin layers of constant index allowing the index profile of the stacks to vary in any function or shape desired to simulate the actual index. The number of layers within each stack depends on how intricate and quickly changing the modeled index changes with respect to the x coordinate.

The B1 stack contains 43 discretized layers where the dielectric variation occurs along the same axis for all 43 layers. Those layers consist of one layer of polarizer, one layer of retarder, and 41 layers of LC. The B2 stack has 2000 discretized layers of hologram media in which the dielectric variation occurs along an axis different from the one above. In this example, the B2 stack also represents the reflector, but this is not necessary to the method, the second stack could contain a mirror or there may not be a reflector in the stack and then this method would be adapted for transmission. The final stack, or B3 stack, is, in this example simply a reversal of the first birefringent stack, B1.

When the B3 stack is defined as the reverse B1 stack one needs to consider, upon reflection, which part of the B1 stack should have light incident on it first. Once again, the Berreman discretization axes shown in Fig. A-3 are considered; therefore, the layer closest to the reflector should have light incident on it first. The discretized layers in the
B1 stack must be inverted so that light propagates through the stack as it would in the lab. The reverse B1 stack includes two parts; the first is due to the reflected axes, \( x^R y^R z^R \), being negative, this indicates that the azimuthal angle, \( \phi^R \), is decreased by some angle when compared with \( \phi \) because \((nx, ny, nz)\) goes to \((-nx, ny, -nz)\) (where \( n \) indicates the optic axis orientation), the second part is that the first layer seen by the reflected light beam is the B1 stack layer closest to the reflector. The layer reversal for the case where the wave vector is along the \( \phi = 0^\circ \) axis is illustrated in Fig. A-10.

Light propagates through the B1 stack and the electric and magnetic fields are then incident on the B2 stack. In the B1 stack the incident wave vector and the x-axis form the input plane of incidence, while the back propagating wave vector and the x-axis form a second plane of incidence where the two planes are not parallel (see Fig. A-10). The back propagating wave vector was reflected from the B2 stack and is transmitted through the reverse B1 stack, or B3 stack. This situation occurs because the reflecting layers below the B1 stack (the B2 stack) are rotated with respect to the B1 stack’s z-axis. Because there is a plane of incidence for the input wave vector and a different plane of incidence for the back propagating wave vector the transforms used to rotate the electric field vector from the B1 stack into the B2 stack cannot be used to transform the reflected electric field vector from the B2 stack back into the B1 stack. The back propagating wave vector, in the B1 frame, is not in the plane created by the incident wave vector and the x-axis; therefore, the initial plane of incidence is not parallel to the backwards propagating plane of incidence. Because these two planes are not parallel (Fig. A-10) the
vector $\hat{\mathbf{S}}_{x^R, y^R, z^R}$ is not parallel to $\hat{\mathbf{S}}_{x^R, y^R, z^R}$; therefore, the angle $\rho$ is not equal to $\rho^R$, as is shown in the above calculations.
FIG. A-10. Schematic of how the reverse B1 stack, or B3 stack, relates to the B1 stack for the case where the wave vector is along the $\phi = 0^\circ$. The arrows indicate the calculation direction. The azimuthal angles in the reverse B1 stack are decreased by $180^\circ$ because of the axes transformation. This layer reversal is illustrated for the specific example discussed in the next section.
APPENDIX B

LCD MODELING VERSION 1.5

B.1 Background

Optical modeling is an important tool for understanding liquid crystal optics. Many display parameters can be investigated by first modeling the LCD, which is much quicker and easier than making several different demonstration displays to compare parameters. Other optical modeling programs exist that can simulate LCDs; however, this project required the ability to simulate dielectric variations in two dimensions due to the need to model slanted holograms. “LCD Modeling” has been designed with this requirement in mind including the extension to the Berreman method, which allows for global dielectric variation in two dimensions.

The “LCD Modeling” code is arranged such that the graphical user interface (GUI), seen by the user, is written in Labwindows CVI 5.0 and the optics calculations are written in Matlab 5.3. The GUI writes all of the parameters defined by the user to “.txt” files that are then read into the optics calculation. The GUI is written in Labwindows CVI 5.0 because it is considerable faster than Matlab, also when programming began Labwindows had the most user friendly graphical user interface. The optics code is written in Matlab 5.3 to take advantage of the simple matrix manipulations. Since the director calculation does not require matrix manipulations it is written in Labwindows CVI 5.0, to utilize the speed advantage due to the C language. The open source code
written by Salman Saeed for the program "GNU-LCM" is used to build from instead of rewriting code that simply defines the optics stack. The basic Labwindows CVI functions surrounding the definition of the optics stack in "GNU-LCM," such as "OpticsPanel.c" and its associated functions, have been expanded in "LCD Modeling."

B.1.1 Director Calculation

The LC director configuration is calculated separate from the Berreman program using a free energy minimization program. The Frank-Oseen free energy density (including an electric field term) is defined as

\[ f = \frac{1}{3} \left( K_{11} \cdot \left( \nabla \cdot \hat{n} \right)^2 + K_{22} \cdot \left[ \hat{n} \cdot \left( \nabla \times \hat{n} \right) - q_0 \right]^2 + K_{33} \cdot \left[ \hat{n} \times \left( \nabla \times \hat{n} \right) \right]^2 \right) - \frac{1}{2} \cdot \left( \tilde{D} \cdot \tilde{E} \right). \]  

(B.1)

where \( K_{11}, K_{22}, \) and \( K_{33} \) are Frank elastic constants, \( \hat{n} \) is the unit vector describing the LC director orientation, \( D \) is the electric displacement, \( E \) is the electric field, and \( q_0 = 2\pi / \rho \), where \( \rho \) is the natural pitch of the LC.

The director orientation is described using standard Spherical and Cartesian coordinates (\( z \) along the cell normal), where the spherical coordinates defined here are different than those defined for the Berreman program. This derivation of the director orientation assumes that the director changes only along the cell normal,

\[ \hat{n} = \sin \theta(z) \cdot \cos \varphi(z) \hat{i} + \sin \theta(z) \cdot \sin \varphi(z) \hat{j} + \cos \theta \hat{k}. \]  

(B.2)

Using Eq. (B.2) we can write the free energy density as:
\[
f' = \frac{1}{2} \left[ K_{11} \sin^2 \theta \left( \frac{\partial \phi}{\partial z} \right)^2 + K_{22} \left[ \sin^4 \theta \left( \frac{\partial \theta}{\partial z} \right)^2 + 2q_o \sin^2 \theta \left( \frac{\partial \theta}{\partial z} \right)^2 + q_o^2 \right] \right] + K_{33} \left[ \cos^2 \theta \left( \frac{\partial \theta}{\partial z} \right)^2 + \sin^2 \theta \cos^2 \theta \left( \frac{\partial \phi}{\partial z} \right)^2 \right] \right] - \frac{1}{2} \cdot (\vec{D} \cdot \vec{E}). \tag{B.3}
\]

The free energy, \( F = \int f \, dz \), is minimized when:

\[
\frac{\partial f}{\partial \theta} - \frac{\partial}{\partial z} \left( \frac{\partial f}{\partial \phi} \right) = 0 \tag{B.4}
\]

\[
\frac{\partial f}{\partial \phi} - \frac{\partial}{\partial z} \left( \frac{\partial f}{\partial \theta} \right) = 0
\]

The free energy minimization is achieved in the program by discretizing the LC layer into 41 layers and then for each layer find the \( \theta \) and \( \phi \) which make the Euler-Lagrange equations zero.
FIG. B-1. Defined axes for the Frank-Oseen free energy density.
B.1.2 Berreman Optics Calculation

The optics calculation is completed using the Berreman 4x4 method that is described in detail in appendix A. The extension of the Berreman method that allows for dielectric variation in two dimensions is also implemented in this program. This allows the Berreman method the added ability to calculate light propagation through slanted holographic layers. This extension allows the slanted layers to not only consist of isotropic layers but also uniaxial layers. The “cmat.m” function called from the optics calculation in “LCD Modeling” was written by Hodgkinson and Wu for Ref 40.

B.1.3 Optics Calculation

The optics calculation written with Matlab 5.3 has many different functions that each complete different calculations, such as; minimizing the difference in polarization state between two points in the display, finding the required polarizer and retarders for a defined input polarization, transmission or reflection versus wavelength, phase shift versus wavelength, phase shift versus incident angle, and viewing angle. Each of these functions are described in the next section, but all of the separate functions have the following general outline.

I. Function Called by User Interface

II. Function Begins

a. Read in the output file name from predefined location

b. Read in the user defined system parameters for particular function

   i. Examples of parameters: $\theta, \varphi$, wavelength, incident material’s index
c. Read in optical stack consisting of required optical material parameters defined by user
   i. Parameters: ne, no, $\psi$, $\xi$, d, location of holograms and LC layers, number of layers
   ii. Function name: makestackelement3ht.m

d. Group optical stack parameters into cells (cells are a matrix of anything for instance it can be a matrix of matrices) to easily call each optical component from program

e. Predefine all cells, matrices, and vectors that will be called in the program as zeros, this saves time during calculation

f. Define hologram parameters
   i. If the hologram is form birefringent, find ne
       1. Define layer normal or form birefringent optic axis as parallel to film normal
       2. Find the phase shift through the form birefringent hologram
       3. Find ne from phase shift, this is described in detail in chapter 3.2.1
       4. Function name: findne.m
   ii. If the hologram is not form birefringent, ne defined as average of n1 and n2
   iii. Define the axes rotation angles $\theta^\text{n}$, $\rho$, $\theta^\text{R}$, and $\rho^\text{R}$, described in detail in chapter A.2
1. Function name: axesrotationview.m

iv. If the hologram is form birefringent, recalculate ne using $\theta''$ instead $\theta$

v. If the hologram is form birefringent, recalculate the axes rotation angles, $\theta'', \rho, \theta^r$, and $\rho^r$, using the redefined ne

vi. If the hologram is form birefringent, calculate the corrected thickness as described in chapter A.2.4

1. Function name: findret.m

g. If there is more then one hologram, complete part f. for each hologram

h. If the function accounts for wavelength dispersion, define the LC and / or retarder's indices of refraction using Cauchy coefficients

i. Calculate an $\tilde{A}$ matrix for each of the holograms, the $\tilde{A}$ matrix corresponds to all the matrices multiplied together in Eq. A.33

\[ \tilde{\psi}_{out} = \tilde{A} \cdot \tilde{\psi}_{enter} \]  \hspace{1cm} (B.5)

j. Define the rest of the optical stack

i. At this point the optical stack can be defined as a separate stack for each optical component, referred to as “separate” in the GUI, or one large optical stack, referred to as “together” in the GUI.

k. Calculate an $\tilde{A}$ matrix for the rest of the optical stacks

l. Input incident electric field

i. For coherent light, the polarization of the incident electric field can be directly defined
ii. For incoherent light, s polarized light is input and the output
transmission or reflection is calculated then p polarized light is
input and a second output transmission or reflection is calculated
and finally the two outputs are averaged together. This is
explained in detail in chapter A.1.2

III. Function Completed and then Outputs Information

B.2 Starting the Program

B.2.1 Coordinate System

The coordinate systems used in the program are the ones defined in Chapter
A.1.3. When using this program the only axes that need to be considered are the lab
fixed coordinate system, which is a right-handed coordinate system that has light
traveling along the positive x direction when the angle of incidence is zero, Fig. A-2.
The other coordinate systems defined in Chapter A.2.1 are implemented inside the
program. The discretized Berreman layers are in the yz plane (for layers that are not
slanted). Each optical component is defined by its optic axis, using its polar and
azimuthal angles, Fig. B-2. The incoming light ray is defined by a different polar and
azimuthal angle. However, both sets of angles are defined correctly for a right handed
coordinate system, meaning that they rotate counter-clockwise.
FIG. B- 2. Berreman axes definition. Each optical component’s optic axis is defined by its polar angle, $\psi$, and its azimuthal angle, $\xi$. The incoming light ray is described by a different polar angle, $\theta$, and azimuthal angle, $\varphi$. 

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B.2.2 Main Window

When the program starts the main window opens. On the main window are ten buttons including the “Exit” button, which ends the program. The top two buttons, “Mid-Layer Theta” and “Optical Stack”, open panels that do not require initialization from other panels in the program. The remaining seven buttons require the “Optical Stack” panel to define the desired optical configuration, those buttons are; “Minimize Polarization Difference,” “Find Retarders,” “T/R vs. Wavelength,” “Phase vs. Wavelength,” “Phase vs. Incident Angle,” and “Viewing Angle.” All panels can be accessed and run using the default optical configuration, in this case the “Optical Stack” panel does not need to be defined first. Fig. B-3 shows the main window of the GUI.
FIG. B-3. LCD Modeling’s main panel.
B.2.3 Mid-Layer Theta Calculation

The “Mid-Layer Theta” panel calculates the LC’s polar angle in the middle of the LC layer with respect to varying voltage. This calculation allows the user to define the non-select and select voltages for the LC material of interest without first defining the optical stack surrounding the LC. Therefore, the values found for the non-select and select voltages are not dependent on the retarders used but are only dependent on the LC parameters. This is particularly useful when designing STN LCDs, especially one polarizer reflective STNs, which require a very complicated retarder stack surrounding the LC, see chapter 5. On the panel there are two categories of variables that must be input, “Cell Parameters” and “Material Parameters.”

The “Cell Parameters” include: “Cell Thickness,” “d/p” (thickness divided by pitch), “Initial Voltage,” “Voltage Step,” “Final Voltage,” “Initial Theta,” “Theta Middle,” “Final Theta,” and “Twist Angle.” The variable “d/p” defines the rotary power of the LC and is negative for a right handed twist and positive for a left handed twist. The polar angle at the top interface of the LC and glass substrate is defined as “Initial Theta” and at the bottom interface is “Final Theta.” The pretilt is a similar variable that is sometimes used in place of the polar angle at the surface, where polar angle = 90°-pretilt. The parameter “Theta Middle” is used as an initial value for the calculation to only decrease calculation time. The parameter “Twist Angle” is the total twist in the LC layer. The “Material Parameters” include; the elastic constants “K_{11},” “K_{22},” and “K_{33},” along with “Epsilon parallel” and “Epsilon perpendicular,” which are the permittivity tensor values for a uniaxial material that is aligned along the material axes. Once the
mentioned variables are defined the “Start” button is pushed and three graphs are calculated; “Mid-Layer Theta vs. Voltage,” “Theta vs. Layers,” where “Layers” is the number of layers through the cell thickness, and “Phi vs. Layers.” The data is saved in the file name defined under “File Name.” This file is located in the Data folder in the LCD Modeling folder. This data file contains the mid-layer polar angle, mid-layer azimuthal angle, and voltage, all separated by a tab. After these three columns the LC’s parameters are listed under the first column.

There is one final calculation that can be completed using this panel, either the “Maximum Voltage,” “Multiplex Ratio,” or “Minimum Voltage.” The calculation is picked using the slide bar labeled “Calculation.” Three variables are shown next to the slide bar, those are; “Maximum Voltage,” “Multiplex Ratio,” and “Minimum Voltage.” The two variables that are not being calculated must be input to complete the calculation. If the desired variables are not input the default values are used. Fig. B-4 illustrates the GUI for the mid-layer theta calculation.
FIG. B-4. Mid-Layer Theta Panel.
B.2.4 Optical Stack Specification

The “Optical Stack” panel allows the user to define the desired configuration of optical components. On the panel are 24 possible optical components called “Elements.” Light enters the optical stack beginning with “Element 1.” The elements can be defined as 25 different optical components. The possible components are; “Top Polarizer,” “Bottom Polarizer,” “Top Glass,” “Bottom Glass,” Aluminum Mirror, 3 different LC layers, 6 different holograms, 10 different retarders, and an empty element. The first LC layer and the first 8 retarders can have wavelength dispersion defined using Cauchy coefficients. The empty element is used at the end of the optical stack for all remaining elements. Fig. B-5 illustrates the GUI used to define the optical stack.

To define each element click on the arrow to the left of the element’s name and a drop down menu will appear, choose the desired optical component for that element. Once the optical stack is defined each optical component needs to have its parameters defined. This is done by clicking on the button entitled “Prop.” to the right of each element. A panel will open that allows the user to define various parameters for the optical element being described. The only element that does not have variable parameters is the Aluminum mirror, its parameters are defined in the file entitled “aluminum.dat” in the Config folder. The parameters in the Aluminum mirror file are the refractive indices real and imaginary components at 550 nm. Fig. B-6 illustrates the window that appears when the mirror element is selected.
FIG. B-5. Optical Stack panel.
FIG. B-6. Aluminum mirror does not require defining any parameters.
B.2.5 Polarizer and Glass Layers

When defining the “Top Polarizer” or the “Bottom Polarizer” the polarizers’ optic axes can be changed. The “Tilt” changes the polarizer’s polar angle of its optic axis. This is usually not varied for standard polarizers. The optic axis is typically in the plane of the polarizer and the default value is 90°. The “Azim.” changes the polarizer’s azimuthal angle of its optic axis; therefore, this parameter varies the absorbing axis of the polarizer. The “Thickness” changes the thickness of the absorbing polarizer layer. The polarizer indices have real and imaginary components and can be input from the static values at the bottom of the panel entitled “No Real,” “No Imag.,” “Ne Real,” and “Ne Imag.” or they can be input from a file which has indices that vary with wavelength. The refractive indices imaginary components make the polarizers absorbing. Fig. B-7 illustrates the GUI used to define the top polarizer parameters. A similar GUI appears for the bottom polarizer.

The “Top Glass” and “Bottom Glass” properties include three layers; glass, indium-tin oxide (ITO), and polyimide. All three layers are isotropic and its refractive index can be changed. The thickness of each layer can also be changed. Fig. B-8 illustrates the GUI used to define the top and bottom glass elements.
FIG. B-7. Top Polarizer parameters.
FIG. B-8. Top glass parameters.
B.2.6 Liquid Crystal Layers

The three liquid crystal layers have several parameters that are used for the director calculation. Similar to the “Mid-Layer Theta” calculation there are “Cell Parameters” and “Material Parameters,” which have the same definitions as above. The “Voltage” variable changes the voltage applied to the LC layer. The “Rotation” variable rotates the entire LC layer. The LC’s indices are defined in the parameters: “no” and “ne”. For only the first LC layer entitled “Liq. Crystal 1,” the LC’s indices can also be defined using the Cauchy coefficients by flipping the switch entitled “Cauchy Dispersion Formula” to “on.” The Cauchy values, shown at the bottom of the panel; “Ao,” “Bo,” “Ae,” and “Be,” can currently only be used with the calculation panels “T/R vs. Wavelength,” “Phase vs. Wavelength,” and “Viewing Angle.” The Cauchy formulas are as follows:

\[
\begin{align*}
n_o &= A_o + \frac{B_o}{\lambda^2}, \\
n_e &= A_e + \frac{B_e}{\lambda^2},
\end{align*}
\]

where \( \lambda \) is the wavelength of incident light in vacuum and in nanometers. Fig. B-9 illustrates the GUI used to define the LC parameters.
FIG. B-9. LC parameters and panel where the director configuration is calculated.
B.2.7 Retarder Layers

The ten retarder layer’s panels contain parameters that define each retarder separately. The uniaxial retarder’s refractive indices is defined using the panel’s parameters; “no” and “ne.” The retarder’s optic axis is defined using the parameters “Polar Angle” and “Azimuthal Angle.” The parameter “Polar Angle” corresponds to the angle $\psi$ shown in Fig. B-2 and the parameter “Azimuthal Angle” corresponds to the angle $\xi$ shown in the same figure. The retarder’s thickness is defined, in microns, using the parameter “Thickness.” The retarder’s resulting retardation is calculated using Eq. (B.7) and that value is output, in nanometers, in the parameter entitled “Retardance.”

$$R = (n_e - n_o)d$$  \hspace{1cm} (B.7)

Where R is the retardance and $d$ is the retarder’s thickness. In the first 8 retarders, the retarder can have several different layers instead of the one layer described using the values provided with the parameters “Polar Angle” and “Azimuthal Angle.” When the switch entitled “Choose Ret. Orientation” is turned to “From File” the retarder’s optic axis orientation is defined by the file given in the box entitled “Retarder File.” The box entitled “Retarder File” is undimmed when the switch is turned to “From File” and the optic axis angle parameters are dimmed. A file can be picked by pressing the “Browse” button. The required file format is one column with the number of layers in the retarder at the top of the column. After the number of layers, the retarder’s polar optic axis angles are listed followed by the azimuthal optic angles. The bottom of the first 8 retarder panels allows the user to define the refractive indices with Cauchy coefficients instead of the static “no” and “ne” above. When the switch entitled “Cauchy Dispersion Formula”
is turned to “on” the Cauchy coefficients below, “Ao,” “Bo,” “Ae,” and “Be,” are used to define the retarder’s indices as is shown in Eq. (B.6). The Cauchy coefficients can currently only be used with the calculation panels “T/R vs. Wavelength,” “Phase vs. Wavelength,” and “Viewing Angle.” When the Cauchy dispersion switch is turned to “in” a modified equation uses the Cauchy coefficients to define the indices:

\[
\begin{align*}
    n_o &= A_o + B_o \cdot \ln(\lambda) \\
    n_e &= A_e + B_e \cdot \ln(\lambda)
\end{align*}
\]  
(B.8)

where, once again, \( \lambda \) is the wavelength of incident light in vacuum and in nanometers. This modified equation was derived to describe the wavelength dispersion of TAC. Fig. B-10 illustrates the GUI used to illustrate the GUI used to define the retarder parameters.
FIG. B-10. Retarder parameters.
B.2.8 Hologram Layers

The 6 hologram panels all have the same parameters. The top half of the panel contains hologram parameters, which define the hologram. The bottom half of the panel contains parameters that define the transmission or reflection versus wavelength calculation (spectra). The spectra calculation is not required to define the hologram. It is available on this panel to allow the user to verify that the hologram, defined in the top half of the panel, is defined correctly.

The hologram parameters "nmax" and "nmin" define the maximum and minimum isotropic refractive indices used in the hologram's dielectric profile. The parameter "Delta n" allows the hologram to be uniaxial, then having an index profile for both the ordinary and extraordinary indices. "Delta n" defines the amount that the extraordinary index is greater then the ordinary index when it is positive and less then when it is negative. In the case that the hologram is uniaxial ("Delta n" something other then 0), the parameters "Polar Angle" and "Azimuthal Angle" define the hologram's optic axis orientation in the exact same way the retarders optic axes are defined. The parameter "Hologram Slant Angle" defines the angle between the hologram's tilted layers and the untilted lab fixed frame (Appendix A.2.1). The "Hologram Slant Angle" is shown in Fig. A-3 (b) as the variable h. The parameter "Total Thickness" is the holograms thickness in the lab fixed frame, which is shown in Fig. A-9 (a) as the variable d. The hologram's peak reflection occurs at the wavelength defined by the parameter "Bragg Wavelength." The parameter "Dielectric Profile" allows the user to choose the functional form of the index variation within the hologram. The possible dielectric profiles are "Sine Wave,"
“Clipped Sine Wave,” “Chirped Sine Wave,” and “Square Wave.” These functional forms and how they affect the hologram’s performance are described in detail in chapters 2 and 4. When the functional form “Chirped Sine Wave” is selected the two parameters “Coefficient A” and “Coefficient B” are no longer dimmed and can now be used. These parameters refer to the amount of chirping that occurs due to the exponential periodicity input into the sine wave profile. This functional form and its affect are discussed in chapter 4.1.2. The parameter “Rotation” rotates the entire slanted hologram stack in the same counter-clockwise direction that retarders rotate when their azimuthal angle is changed. The switch “Form Birefringence” allows the user to define the hologram’s periodicity with the parameter “Layer Thickness”, which is undimmed when the switch is turned “on.” When the periodicity is defined using “Layer Thickness” the parameter “Bragg Wavelength” is dimmed since this parameter is used to define hologram periodicity when the switch “Form Birefringence” is turned “off.” The hologram parameters are defined in the optical configuration by clicking the “Done” button. To continue on the hologram panel and graph the hologram’s spectra the “Done” button is not clicked but the necessary spectra parameters are defined and the “Graph Spectra” button is clicked instead. Fig. B-11 illustrates the GUI used to define the hologram parameters.

The spectra calculation parameters define the variables used to calculate the hologram’s spectra. The minimum wavelength, maximum wavelength, and wavelength step size must be defined along with the wavelength that the data is averaged over. The wavelength averaging corresponds to the coherence length of the incoming light, as is
discussed in appendix A.1.2. The parameter “Incident Angle” defines the incoming light’s polar angle, shown as the variable $\theta$ in Fig. B-2. The parameter “Inc. Azimuthal Ang.” defines the incoming light’s azimuthal angle, shown as the variable $\phi$ in Fig. B-2.

The variables “Theta” and “Phi” define the incoming light’s polarization state using the Poincare sphere angles defined in chapter 5. The variable “Theta” is shown in Fig. 5-9 $2\omega$ and the variable “Phi” is shown in the same figure as $2\alpha$. The refractive index of the material surrounding the hologram is defined using the variable “Input Index, nin.”

Either the reflection from the hologram or transmission through the hologram can be graphed by choosing “Reflect” or “Transmit”, respectively, on the switch entitled “Reflection or Transmission.” After all the above parameters are defined the hologram’s spectra is calculated by clicking the “Graph Spectra” button, resulting in two graphs. The first graph shows the hologram’s spectra, and the second graph shows the hologram’s index profile or Index versus hologram thickness. Once the user is finished with these graphs they are exited and the hologram panel can also be exited by clicking the “Done” button. The data resulting from the spectra calculation is found in the file entitled “holospectra.txt” and is found in the Config folder.
FIG. B-11. Hologram parameters.
B.3 Various Data Calculations

After the optical stack is defined any of the available calculations can be completed. Parameters that are common to several panels will now be described. Similar to the hologram spectra calculation, the parameter “Incident Angle” defines the incoming light’s polar angle, shown as the variable $\theta$ in Fig. B-2. The parameter “Inc. Azimuthal Ang.” defines the incoming light’s azimuthal angle, shown as the variable $\varphi$ in Fig. B-2. The variables “Theta” and “Phi” define the incoming light’s polarization state using the Poincare sphere angles defined in chapter 5. The variable “Theta” is shown in Fig. 5-9 as $2\omega$ and the variable “Phi” is shown in the same figure as $2\alpha$. The refractive index of the material surrounding the optical stack is defined using the variable “Input Index, nin.” The incident angle is always in vacuum the program takes the value input into the GUI and uses the input incident index, “Input Index, nin,” along with Snell’s law to calculate all the angles used in the program. The program is designed this way so that a change of incident index does not change the incident angle with respect to vacuum. The switch entitled “Reflection or Transmission” allows the user to choose to calculate either the reflection or transmission. The Berreman method can calculate both transmission and reflection at the same time but when one component in the optical stack is slanted (hologram) only one or the other can be calculated. This is due to coordinate transformation discussed in appendix A, which includes discarding some of the forward and reverse propagating fields, shown in Fig. A-5. The switch entitled “Stack” defines if all optical components are in the same stack, “Together,” or if each optical component has its own optical stack, “Separate.” To calculate light propagation through slanted
layers each component must have its own optical stack so that the total electric fields are calculated before and after each component, allowing coordinate transformations to occur.

B.3.1 Minimize Polarization Difference

The “Minimize Polarization Difference” panel calculates the polarization difference between two locations in the optical stack. This program was mainly used to complete the calculations needed for the single polarizer STN configuration in chapter 5. This program is very useful when the entire optical stack has not been completely defined but the user wants to optimize the input polarization state without creating an input retarder stack along with optimizing the LC’s thickness, rotation angle, and non-select voltage. This program outputs the angular difference between the input polarization state and output polarization state, where both states are represented on the Poincare sphere and the angular difference between the two states is the angular distance on the Poincare sphere. The angular distance between the input and output polarization states is defined as gamma. Gamma can be calculated while varying several different parameters, such as; input polarization state, LC thickness, rotation angle, and non-select voltage, all or only a few of these parameters can be varied at the same time. The slide bar entitled “Find:” allows the user to choose the optimization completed. When “Maximum Gamma” is selected only the gamma values above the value defined in the parameter box called “Limit” are output. Once the gamma values are calculated for the select state, the program automatically takes the input parameters that correspond to the maximum gamma values and calculates the gamma values for the non-select state. If “All Gamma”
is selected instead, all variable permutations are calculated for only the select state and no calculations are completed with the non-select state. If “Minimum Gamma” is selected, all variable permutations are calculated for only the non-select state and no calculations are completed with the select state. The button entitled “Generate NS Dir. Files” opens another director configuration file that allows a series of director configuration files to be calculated which differ in voltage by increasing steps. There are default director files that already exist that can be used for the calculation. The output data files for both the select and non-select voltages is defined as the respective parameters: “Output Select State File” and “Output Non-Select State File.” The output select and non-select files each have 4 columns separated by tabs. The columns are the calculated angular difference, corresponding retardation divided by wavelength, and corresponding input polarization angles $2\omega$ and $2\alpha$. Fig. B-12 illustrates the GUI used to define the parameters needed for the polarization differences calculation. Fig. B-13 illustrates the GUI used to define the non-select LC parameters.
FIG. B-12. Minimize Polarization Difference panel.
FIG. B-13. Generate non-select director configuration panel.
B.3.2 Find Retarders

The "Find Retarders" panel calculates the angular difference between the calculated output polarization and the input desired output polarization defined on this panel with the parameters "Desired Theta of output light" and "Desired Phi of output light." This program is used to find the retarder stack that will best create the required polarization state found using the "Minimize Polarization Difference." The input polarization outside the retarder stack is defined and the desired output polarization state, which was found using the previous program, is also defined. In the retarder stack, the thickness of one of the optical components can be varied or two separate components can be rotated to find the optimum configuration. The parameter "Linear Polarization Output?" forces the desired output polarization to be linear when "Yes" is selected. This is done by setting the desired output polar angle, $2\omega$, is equal to 90° and the desired output azimuth angle, $2\alpha$, is equal to the calculated output polarization azimuthal angle; therefore the angular distance measured is between the calculated output polarization and the equator of the Poincare sphere. The output file name is defined in the box called "Output File." The resulting output file has the calculated gamma values for the corresponding variable. Fig. B-14 illustrates the GUI used to define the parameters needed for the find retarders calculation.
FIG. B-14. Find Retarders panel.
B.3.3 Transmission or Reflection versus Wavelength

The “T/R vs. Wavelength” panel calculates either the transmission or reflection from the display with respect to varying wavelength. This program is used for almost any display that is considered. One component in the display can be rotated using the drop down menu entitled “1st Component Rotated.” The angles through which the component is rotated are defined below in the variables defined “Starting Rotation 1,” “Rotation Step 1,” “Stop Rotation 1.” This feature was created to more quickly optimize transmissive STN. A transmissive STN generally uses a full waveplate to make the dark state more achromatic. The full waveplate is rotated until the most achromatic dark state is found. The output file is defined in “Output File” and can be found in the “Data” folder. The resulting output file has two columns; first the transmission or reflection and then the wavelength. At the head of the column is the rotation angle of the component that is rotated, if there is one. The columns are separated by tabs. Fig. B-15 illustrates the GUI used to define the parameters needed to define the transmission or reflection versus wavelength calculation.
FIG. B-15. Transmission or Reflection versus Wavelength panel.
B.3.4 Phase Difference versus Wavelength

The “Phase vs. Wavelength” panel calculates the phase retardation resulting from the optical stack defined in the optics panel. The calculation is used to find the retardation of the holographic layers in chapters 2 and 3. It is especially useful when matching the wavelength dispersion of the hologram to that of a particular LC material, as done in Chapter 3. This calculation is defined so that the optical stack is automatically surrounded by ideal, crossed polarizers. This is necessary because of the nature of this calculation, to find a phase difference the initial phase must be either known or zero and in this case it is zero. The transmission axis of the ideal polarizer is oriented at +45° and the ideal analyzer is at -45°. Both of these are angular independent because Jones matrices are used. The ideal polarizer is created by making the input polarization state, \( P \), the Jones vector for +45° polarized light\(^{47} \):

\[
P_{\text{in}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.
\]  \hspace{1cm} (B.9)

The ideal analyzer is created by multiplying the output polarization state by the Jones matrix, \( J \), for an ideal homogeneous linear polarizer with its transmission axis at -45°:

\[
P_{\text{final}} = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \cdot P_{\text{out}}.
\]  \hspace{1cm} (B.10)

The output file name is defined in “Output File” and can be found in the “Data” folder. The resulting output file has two columns of data separated by tabs. The first column is the wavelength and the second column is the calculated phase difference. Once all the phase difference values are shown a second set of data follows, in which the first column is again the wavelength and the second column is the reflectance. The second set of data
is of interest because one must make sure that the form birefringent retarder is not in the Bragg regime. Fig. B-16 illustrates the GUI used to define the parameters needed for the phase difference verses wavelength calculation.
FIG. B-16. Phase Difference versus Wavelength panel.
B.3.5 Phase Difference versus Incident Angle

The "Phase vs. Incident Angle" panel calculates the phase retardation resulting from the optical stack with respect to varying incident angle. This calculation is used to find the phase retardation of the form birefringence retarders at varying incident angles. It allows the user to be sure that the retarder is well out of the Bragg regime at all angles and that the retarder behaves like a linear retarder. This calculation is also defined so that the optical stack is automatically surrounded by ideal, crossed polarizers, using Eqs. (B.9) and (B.10). The output file name is defined in "Output File" and can be found in the "Data" folder. The resulting data is in three columns separated by tabs. The first column is the incident angle in vacuum, then the reflectance, and finally the phase difference. Fig. B-17 illustrates the GUI used to define the parameters needed for the phase difference verses incident angle calculation.
FIG. B-17. Phase Difference versus Incident Angle panel.
B.3.6 Viewing Angle

The "Viewing Angle" panel calculates either transmission or reflection for varying polar and azimuthal angles. This calculation is used to find the isoluminance plots for both the bright and dark state of the display. The transmission or reflection can be calculated for several wavelengths that are then averaged together. The output file name is defined in "Output File" and can be found in the "Data" folder. The resulting data is in four columns separate by tabs. The columns are; wavelength, polar angle, $\theta$, azimuthal angle, $\phi$, and transmission or reflection. A second file is also output that is in the format required for "Contour Plotting" program written by Chad Hoke. The second file is always called "viewingangle.prm" and is found in the "Config" folder. This second file has four columns that are separated by fixed width. The columns are; wavelength, polar angle, azimuthal angle, and transmission or reflection. This second file is created so viewing angle plots can be easily generated. A button entitled "Contour Plot" links to the "Contour Plotting" program. The output files for the bright state and dark state can be used to find the contrast ratio. To plot the contrast ratio on a viewing angle plot the resulting data must be put into a "*.prn" file with the above described format. At this time the only polar and azimuthal angles that will be plotted correctly with the "Contour Plotting" program are the default values; $\theta = 0^\circ$ through $60^\circ$ in steps of $10^\circ$, and $\phi = 0^\circ$ through $348^\circ$ in steps of $12^\circ$. Fig. B-16 illustrates the GUI used to define the parameters needed for the viewing angle calculation.
FIG. B-18. Viewing Angle panel.
B.4 Required Folder Locations

The main folder for the program entitled “LCD Modeling” must be located in the C:\ drive. All folders associated with the program are located in the main folder and are “Config,” “Contour Plot,” “cvi-code,” “data,” “matlab code,” and “non_selectDir.” The “Config” folder has required variable files. The “Contour Plot” folder has the “Contour Plotting” program written by Chad Hoke. The “cvi-code” folder has all the files associated with the GUI of the program, which were written in Labwindows CVI 5.0. The “data” folder has the resulting data files. The “matlab code” folder has all files associated with running the Berreman optics code, which were written in Matlab 5.3 and 6.5. The “non_selectDir” file has all director files that were created for the non-select state for the “Minimize Polarization Differences” panel. All other director files are in the “Config” folder.

B.5 Examples and Verifications

Many verifications and tests were completed as the program was being written. The results presented in chapter 4 Fig. 4-4 shows that when modeling holographic reflectors the Berreman calculation agrees well with the coupled wave theory calculation, which is the typical method used to model holograms. The calculated and measured reflections from both a slanted hologram alone and a slanted hologram with an LCD in front of it are compared in Figs. 4-9 and 4-10 and the two results have very good agreement. The modeling of holographic retarders is verified by comparing holographic retarders with their corresponding negative birefringence uniaxial retarder. Polar plots showing the transmission through either the negative uniaxial retarder or holographic
retarder between crossed polarizers are in Figs. B-19 through B-24. The polarizer transmission axis is at 0° and the analyzer’s transmission axis is at 90°. The parameters of the uniaxial retarder are \( n_e = 1.85 \), \( n_o = 1.75 \) and \( d = 1.11 \ \mu \text{m} \). The optic axis is parallel to the film normal in Fig. B-19. The hologram modeled was mentioned in chapter 2 that matches that investigated by Eblen and associates\(^{29}\), where \( n_1 = 2.13 \) and \( n_2 = 1.53 \). The layer thickness is 20 nm (40 nm periodicity) and the total thickness of the hologram is 1.11 \( \mu \text{m} \). The layers are unslanted and parallel to the surface of the hologram in Fig. B-20. Notice that there is very good agreement between the negative, uniaxial retarder and the holographic retarder. Fig. B-21 shows the transmission through a negative, uniaxial retarder with its optic axis 20° from the film normal between crossed polarizers. Fig. B-22 shows the transmission through a slanted holographic retarder with its layer normal 20° from the film normal between crossed polarizers. Once again, there is very good agreement between the transmission polar plot of the negative, uniaxial retarder and that of the holographic retarder. Fig. B-23 shows the transmission through two negative, uniaxial retarders between crossed polarizers. The first retarder’s optic axis defined by \( \psi = 10° \) and \( \xi = 10° \) and the second retarder’s optic axis is defined by \( \psi = 44.75° \) and \( \xi = 30° \). Fig. B-24 shows the transmission through two slanted holographic retarders between crossed polarizers. The first holographic retarder’s layer normal is defined by \( \psi = 10° \) and \( \xi = 10° \) and the second holographic retarder’s layer normal is defined by \( \psi = 44.75° \) and \( \xi = 30° \). Notice that at \( \theta = 60° \) and \( \varphi = 180° + \xi \), there is a slight deviation between the holographic retarder and uniaxial retarder. This deviation occurs because of the highly slanted second holographic retarder. For highly slanted
holograms, at an incident angle inside the holographic material of 90°, light propagates along the layers. This is a limitation of the method devised for slanted layers.
FIG. B-19. Transmission through a negative, uniaxial retarder between crossed polarizers. The parameters of the uniaxial retarder are $n_o = 1.85$, $n_e = 1.75$, $d = 1.11 \, \mu m$, the optic axis is parallel to the film normal.
FIG. B-20. Transmission through a holographic retarder between crossed polarizers.

The parameters of the holographic retarder are $n_1 = 2.13$, $n_2 = 1.53$, $d = 1.11 \, \mu m$, the layers are unslanted.
FIG. B-21. Transmission through a negative, uniaxial retarder between crossed polarizers. The optic axis is 20° from the film normal.
FIG. B-22. Transmission through a holographic retarder between crossed polarizers.

The layers are slanted so that the layer normal is $20^\circ$ from the film normal.
FIG. B-23. Transmission through two negative, uniaxial retarders between crossed polarizers. The optic axis of the first retarder is $\psi = 10^\circ$ and $\xi = 10^\circ$. The optic axis of the second retarder is $\psi = 44.75^\circ$ and $\xi = 30^\circ$. 
FIG. B-24. Transmission through two holographic retarders between crossed polarizers.
The first holographic retarder has the layers slanted so that the layer normal is defined by
\( \psi = 10^\circ \) and \( \xi = 10^\circ \). The second holographic retarder has the layers slanted so that the
layer normal is defined by \( \psi = 44.75^\circ \) and \( \xi = 30^\circ \).
APPENDIX C

POLARIZATION STATE COMPARISON

C.1 Muller Matrix Proof

A passive birefringent layer does not change the relative distance between polarization states on the Poincare sphere, which will be shown using Mueller matrices. This is a concern when optimizing a display using polarization states, as done in chapter 5.2, where the relative distance, on the Poincare sphere, between polarization states P1 and P4 is equal to the relative distance between polarization states P2 and P3, Fig. C-1, which is not immediately apparent. To simplify this explanation we will create a simplified optical stack (Fig. C-1) that contains a polarizer, quarterwave plate, and mirror.

Without loss of generality, the polarization at p1 can be represented by the normalized, linear Stokes polarization vector:

\[ p_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \]  \hspace{1cm} (C.1)

The Mueller matrix that describes a quarterwave plate with it’s fast eigenvector’s azimuthal angle at 0° is: 47, 48

250
\[ M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}. \tag{C.2} \]

This matrix is defined as the retardance matrix. Any rotation of the retarder can be found using the following rotation matrix \(^{47,48}\),

\[ T(2\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\alpha) & \sin(2\alpha) & 0 \\ 0 & -\sin(2\alpha) & \cos(2\alpha) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{C.3} \]

where \( \alpha \) is the rotation angle of the retarder's fast eigenvector in the lab. For this proof, the problem will be simplified even more by using a lossless, isotropic mirror \(^{49}\):

\[ M_m = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \tag{C.4} \]

The \( M_m \) matrix contains a negative one in the s3 component which accounts for the time reversal operator, light now travels in the opposite direction, and a negative one in the s2 component which accounts for the reciprocity operator, the axes are rotated such that the wave vector is still defined in a right-handed coordinate system and is propagating in the positive direction \(^{50}\). Upon reflection, the optical stack components are described in the new set of axes by completing an axes transformation using the reciprocity matrix, \( O \):

\[ O = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{C.5} \]
The optical components are described in the reflected trip by:

\[ M_{\text{reflected}} = O \cdot M^T \cdot O^{-1}, \quad (C.6) \]

where \( O^{-1} = O \), and \( M \) is by definition an orthogonal matrix. Not only is an axes transformation completed but the principle of reciprocity imposes a transpose on \( M \).

Using the matrices above one can find the Stokes vectors for the polarizations illustrated in Fig. C-1.

\[
\begin{align*}
p2 &= T(-2\alpha) \cdot M \cdot T(2\alpha) \cdot p1 \\
p3 &= M_m \cdot p2 \\
p4 &= O \cdot \left( T(-2\alpha) \cdot M \cdot T(2\alpha) \right)^T \cdot O^{-1} \cdot p3
\end{align*}
\]  

(C.7)

Solving for the above polarizations:

\[
\begin{align*}
p2 &= \begin{bmatrix} 1 \\
\cos^2(2\alpha) \\
\cos(2\alpha)\sin(2\alpha) \\
\sin(2\alpha) \end{bmatrix} \\
p3 &= \begin{bmatrix} 1 \\
\cos^2(2\alpha) \\
-\cos(2\alpha)\sin(2\alpha) \\
-\sin(2\alpha) \end{bmatrix} \\
p4 &= \begin{bmatrix} 1 \\
\cos^2(2\alpha) - \sin^2(2\alpha) \\
-2\cos(2\alpha)\sin(2\alpha) \\
0 \end{bmatrix}
\end{align*}
\]  

(C.8)

To compare the polarization states in the forward propagating direction with the polarization states in the reverse propagating direction the reverse polarization vectors must first be rotated back into the forward propagating axes frame using the reciprocity matrix, \( O \).
\[ p_{N_{\text{forward}}} = O \cdot pN \]  
(C.9)

The variable N defines the polarization vector number for any reverse propagating polarization vector. This results in p3 and p4 which can be contrasted with the forward propagating polarization vectors:

\[
p^{3\text{ forward}} = \begin{bmatrix}
1 \\
\cos^2(2\alpha) \\
\cos(2\alpha)\sin(2\alpha) \\
-\sin(2\alpha)
\end{bmatrix}
\]  
(C.10)

\[
p^{4\text{ forward}} = \begin{bmatrix}
1 \\
\cos^2(2\alpha) - \sin^2(2\alpha) \\
2\cos(2\alpha)\sin(2\alpha) \\
0
\end{bmatrix}
\]

The distance between any two points on the Poincare sphere is the radius of the sphere times the angular distance between the two points. Since the polarization vectors are normalized the distance between any two points on the Poincare sphere is just the angular distance:

\[
\cos(\gamma) = \sin(\theta_1)\sin(\theta_2) + \cos(\theta_1)\cos(\theta_2)\cos(\varphi_1 - \varphi_2). 
\]  
(C.11)

The traditional angles defined for the Poincare sphere are:

\[
2\omega = \sin^{-1}(S_3) \\
2\alpha = \tan^{-1}\left(\frac{S_2}{S_1}\right). 
\]  
(C.12)

Using these definitions one can find the angles for each polarization, for p1:

\[
2\omega_1 = 0 \\
2\alpha_1 = 0
\]  
(C.13)

for p2:
\[ 2\omega_2 = 2\alpha \]  
\[ 2\alpha_2 = 2\alpha \]  
\[ 2\omega_3 = -2\alpha \]  
\[ 2\alpha_3 = 2\alpha \]  
\[ 2\omega_4 = 0 \]  
\[ 2\alpha_4 = 4\alpha \]  

(C.14)

(C.15)

(C.16)

(C.17)

for p3:

and for p4:

Using the above defined angles one can find the angular distance between p1 and p4, \( \gamma_{14} \), and the angular distance between p2 and p3, \( \gamma_{23} \):

\[ \gamma_{14} = \gamma_{23} = 4\alpha . \]

Here it is seen that the relative angular distance between p1 and p4 is equal to the relative angular distance between p2 and p3.
FIG. C-1. Simplified optical stack to aid in showing that the passive birefringent layer does not change the relative distance between polarizations P1-P4 and P2-P3.
## REFERENCES


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