TUNABLE LIQUID CRYSTAL ETALON AND PHOTONIC DEVICES

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To my family.
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Chapter 1

Introduction and background

1.1 Liquid Crystals and Light Propagation in Liquid crystals

Liquid crystals by its very name suggest are very unique group of materials that exhibit phase behaviors of both liquids and solids. More specifically this so called mesogenic materials has some degrees of orientational and positional order that are characteristics of crystalline solids at the same time exhibiting fluid like disorder. By far the most well understood and in some ways the simplest among the family of liquid crystalline phases is a nematic liquid crystal (LC). Nematogens or the molecules that exhibit nematic phase are usually elongated rod like molecules that in the bulk have a degree of orientational ordering which is described by unit vector called the director (See Figure 1.1).
Macroscopic anisotropy of nematic liquid crystal is a result of orientational ordering of rod-like molecules.

As a result of this macroscopic anisotropy, electromagnetic fields interact with the LC differently depending on the direction of the fields with respect to the director axis. For example, incident electromagnetic field experiences ordinary (perpendicular to the director axis) or extraordinary (parallel to the director axis) or the mix of the two indices of refraction depending on the relative orientation of the field vector and the director axis (See Figure 1.2).

Figure 1.2: Electric field experiences different indices of refraction depending on its orientation with respect to the director axis (for clarity magnetic field vector is not shown)
Similarly strong dielectric anisotropy of the LC molecules combined with the fluidity of the material cause the molecules to easily be reoriented with the application of external electric field. Although there are many other magnificent properties of LC material both from physics and applications point of view, this electro-optic response of LC material perhaps is the most readily exploited property in LC based device technology.

1.2 Liquid Crystal Displays

Since the invention of the first dynamic light scattering LC display (LCD) device by George Heilmeier\textsuperscript{3} at RCA in 1968 and subsequently the discovery of twisted nematic (TN) display by James Fergason\textsuperscript{4} at the Liquid Crystal Institute in 1969 there has been tremendous growth for LCD technology both in terms of device performance and the market size. Although LCD technology has matured and is established as the most prominent display technology for high performance flat panel information display\textsuperscript{5} there are still number of problems with the existing technology. Particularly in the field of mobile displays where the low power consumption is of vital requirement, inefficient light throughput of current LCDs have put the LCD technology in great disadvantage. As the mobile electronic devices such as ipods, laptops, and electronic books, etc witness rapid growth and the demand for higher performance display persists it is no doubt that there will be much more changes imminent in the information display technology landscape.
1.3 Problems with the Existing Technology

Although different modes of LCD such as TN, STN (super twisted nematic), VA (vertical alignment), and IPS (in plane switching), etc have their own advantages in terms of viewing angle, switching speed, and contrast ratio the underlying technology for all of them is based on modulation of polarized light to generate color pixels using absorbing color filters. The maximum theoretical light throughput of two polarizers (<40%) and side-by-side color filters (<33%) amount to less than ~13% (see Figure 1.3) which is unacceptable for high efficiency low power displays.

![Diagram of light throughput components](image)

Figure 1.3: Light throughput of current LCDs is less than 15% as a result of polarizers and absorbing color filters

In remedy to the shortcomings of the LCDs in mobile applications transflective display technology where the pixels are divided into transmissive and reflective counterparts is invented to make the LCDs suitable for applications where ambient light condition
greatly varies\(^8\). Yet constant miniaturization and ever-growing stringent requirement for reduced power use are fueling the need for new display technologies that can compliment the accelerating growth and spread of mobile electronics devices.

### 1.4 Alternative Technologies for Mobile Devices

In this section we will attempt to quickly review the emerging technologies that have promising potential for new low power mobile display applications\(^9\). While technologies such as cholesteric display is still under developmental stage, some of these technologies have already achieved significant milestones in capturing special niche applications of mobile displays such as electrophoretic displays for e-book readers.

The interferometric modulator (IMOD) display currently being developed and marketed by Qualcomm is essentially an optically resonant MEMS device that consists of reflective membrane suspended on a thin film stack\(^{10,11,12}\). By tuning the thickness of the air gap using electrostatic forces incident light is either reflected or absorbed and the gray scale is generated using pulse width modulation (see Figure 1.4)\(^{13}\). The inherently bistable and polarization independent structure of IMOD device makes it a good candidate for low power, high brightness mobile display applications. However, currently the technology offers low resolution bichrome displays only and the development of a full color IMOD display, which requires side-by-side pixel structure, is still underway.
Figure 1.4: Incident light is either reflected or absorbed depending on the air cavity thickness in the iMoD display

Electrophoretic display generates images by manipulation of charged particles by an external electric field\textsuperscript{14,15,16,17}. Figure 1.5 illustrates the working mechanism of microencapsulated electrophoretic display developed by E-Ink Corp (popularized by Amazon Kindle e-book reader). Although the low power consumption and paper-like qualities of electrophoretic display makes it an ideal for ebook readers such as Kindle, slow response time and lack of color prevents its usage in most image intensive personal electronic devices.
Another reflective display technology is cholesteric LC displays\textsuperscript{18,19,20,21}. As Figure 1.6 shows a cholesteric display operates between two bistable zero field states which are reflecting planar texture and transmissive focal conic texture (an absorbing backplane creates the dark state). Although the use of polarizers and the slow switching time result in low-brightness still image displays, their ability to be manufactured in roll-to-roll process, and potential use as flexible plastic display makes cholesteric display very unique in mobile display applications\textsuperscript{22}. The stackable design of cholesteric display makes it one of the best candidates for ultimate low power, reflective, color display for e-paper applications (See Figure 1.7).
Figure 1.6: Pixel structure and working principle of E-Ink's microencapsulated electrophoretic display

Figure 1.7: Cholesteric displays can be made stackable which means that the color pixels can be in series resulting in 3X the brightness gain in comparison to parallel pixel displays
1.5 Overview of the dissertation

It is clear that the drawbacks of the current LCD technology for mobile applications require innovations for new technology that can result in higher brightness, low power, and small form factor displays of tomorrow. In this dissertation we will not attempt to offer the solution to this grand problem. In fact the dissertation is not about display device in particular, but rather a collection of ideas for a new type of LC optical modulators that can have applications in display devices.

Chapters 2 through 4 will cover the polarization independent LC etalon effect and its applications in transmissive field-sequential pico-projection devices. More specifically Chapter 2 will cover the basis for polarization independent LC etalon requirements including the tuning of twist angle and etalon cavity for select wavelengths that are polarization independent. Chapter 3 then will introduce the idea of projection device based on the polarization independent LC etalon and will show the device optimization in terms of etalon order, mirror reflectivity and the LC birefringence. Subsequently the whole of Chapter 4 will cover the design and assembly of a demo device based on the previous two chapters. The challenging set of problems presented by sub-micron cell fabrication and assembly as well as the demonstration of polarization independent wavelength tunability and the interesting solution thereof both from the process and design point of view will hopefully be useful for others who venture to build LC devices that are on the same thickness order.
Chapter 5 will cover design and numerical simulations of a reflective LC etalon device. We will first show simple quarter-wave stack design with switchable LC cavity in the middle and then more complicated design with integrated half-wave dielectric layers will be presented. By tuning the reflectivity of the stacks surrounding the two half-wave layers we will be able to demonstrate high reflectivity, high contrast select spectrum with 30nm bandwidth. Finally, we will optimize the reflective LC etalon device to be polarization independent and will show its possible application as color pixel of a reflective display.

Chapter 6 is a short chapter on tunable LC Bragg grating design. H-PDLC Bragg grating consisting of alternating layers of LC and polymer rich regions can show strong reflection of certain color depending on the periodicity of the stack. Currently tuning range of the reflected light in H-PDLCs is limited by the LC birefringence. We will show that tunability can be increased by having higher order active LC layers with respect to that of the passive polymer layers.

Chapter 7 covers the LC order and switching properties studied in sub-micron LC cells. The original intention of the study was to probe the viability of fast switching LC cells on the order of ~0.5µm for etalon device application. However, we pushed the study further to include experimental probes of aforementioned properties in LC cells with thickness as low as few tens of nm.

In summary, the overall picture of the dissertation can be illustrated by a simple tree diagram shown in Figure 1.8. We see the works on polarization independent LC
etalon effect and the thin cell studies as the fundamental building block that resulted in high efficiency transmissive and reflective devices with sub-micron cell cavity. The applications of these devices can be found in field sequential pico-projection device and in high reflectivity displays.

Figure 1.8: Dissertation outline in tree diagram format
Chapter 2

Polarization Independent Liquid Crystal Etalon

2.1 Introduction

Nematic liquid crystal (LC) is characterized by an orientational ordering which is defined by a unit vector (the director) along the average direction of the elongated molecules’ long axis\(^2\). Orientational ordering of LC results in macroscopic optical and electrical anisotropy which is exploited in variety of applications of electro-optical devices\(^23\). A typical LC Fabry-Perot Interferometer (FPI) consists of two parallel reflecting mirrors sandwiching an optical cavity filled with tunable LC material. If the FP mirrors are identical, then the transmission can be calculated as,

\[
T = \frac{\tau^2}{(1 - \rho)^2 + 4 \rho \sin^2(\delta)}
\]

where, \(\tau\) and \(\rho\) are transmittance and reflectance of the mirrors, and \(\delta = \frac{2 \pi n d}{\lambda} + \phi\) with \(n\), \(d\) and \(\phi\) being the cavity index, cavity thickness and phase change on reflection respectively\(^24\). In this case the resonance condition for transmission maximum is met when \(\delta = m \pi\), \(m = \pm 1, 2, 3\) with \(m\) being the order of the transmission maximum. When the FP cavity is filled with homogeneous non-twist LC, the eigenmodes of light propagation are planar polarized waves parallel (extraordinary mode) and perpendicular (ordinary mode) to the director and the FP transmittance is sum of transmittances of these modes. In general transmission maximums of ordinary and extraordinary modes do not overlap and only the latter one can be tuned with the application of an external field across the FP cavity. Such polarization-dependent
tunability is not desirable in wavelength selection applications, because of losses and polarization fluctuations in real devices. However, as the twist of LC director increases, eigenmodes diverge from planar polarized waves to elliptically polarized modes. In general as the twist angle increases, eigenmodes become more circularly polarized and eventually hits completely circular polarized waves when the wavelength is much greater than the pitch of the director twist.

Poincare sphere\textsuperscript{25,26,27} representation of polarization states of light as it traverses through birefringent optically active media is visually illustrative in probing the evolution of the eigenmodes in twist LC structure. Stokes parameters of the eigenmodes can be plotted on the Poincare sphere with the radius of the sphere being the amplitude of the wave. Each point on the sphere represents a unique polarization state with north and south poles representing right and left circularly polarized lights. It can be found in Bigelow and Kashnow's paper that the ellipticity of the eigenmodes is calculated from the Poincare sphere representation as, \( \omega = \arctan\left[\frac{2d}{\lambda (pu)}\right] / 2 \) where, \( d, p \) are thickness and pitch of the LC, and \( u = \Delta n d / \lambda \).\textsuperscript{28} Here, the ellipticity, \( \omega \), is related to the long and short axis of the polarization ellipse as \( \tan(\omega) = b / a \). Figure 2.1 shows the evolution of eigenmodes as the twist angle of LC cell increases from 0 to infinity. Eigenmodes start as planar polarized waves parallel and perpendicular to the director axis (±S1 on the Poincare sphere) and approaches right and left circularly modes as the twist angle increases (±S3 on the Poincare sphere).
When calculating the state of polarization and the intensity of the light as it travels through optically active birefringent media, it is convenient to represent electric vectors in terms of Jones vectors and optically active mediums as 2x2 matrices called the Jones matrices. If the long and short axis of the polarization ellipse are known, then the Jones vector representation of orthogonal electric fields are \( \vec{E}_1 = (a, -ib) \) and \( \vec{E}_2 = (-ib, a) \). We can let \( |\vec{E}|^2 = a^2 + b^2 = 1 \) in which case \( a = \sqrt{1/(\tan^2 \omega + 1)} \), and \( b = a \tan(\omega) \). Twisted LC structure can be thought of as an infinitesimally small N number of birefringent slabs each placed in \( \phi/N \) angle with respect to its neighboring slab. In this case Jones matrix representation of an individual birefringent slab is given by Eq. (2.1).
Here, \( \Gamma_{\omega,\phi} = 2\pi n_{\omega,d}/(\lambda N) \). The rotated Jones matrix in the lab frame becomes
\( M' = MR(\varphi/N) \) where \( R(\varphi/N) \) is a rotation matrix given by Eq. (2.2).

\[
R(\varphi) = \begin{bmatrix}
\cos (\varphi/N) & \sin (\varphi/N) \\
-\sin (\varphi/N) & \cos (\varphi/N)
\end{bmatrix}
\] (2.2)

Output electric vectors in the rotating frame are calculated using Eq. (2.3).

\[
\left( \vec{E}' \right)_{out}^{1,2} = (M')^N \vec{E}_{in}^{1,2}
\] (2.3)

Electric vectors can be rewritten in the lab frame by multiplying with a rotation matrix (see Eq. (2.4)).

\[
\left( \vec{E} \right)_{out}^{1,2} = R(-\varphi)\left( \vec{E}' \right)_{out}^{1,2}
\] (2.4)

Note that in general electric vectors are of the form \( \vec{E} = (E_x, \exp (i\Delta \theta), E_y) \) where \( \Delta \theta \) is the phase lag of \( E_x \) relative to \( E_y \) \( (\Delta \theta = \theta_x - \theta_y) \) and is constant in the rotating frame. Then the phase difference between input and output vectors of the two eigenmodes is calculated using Eq. (2.5).

\[
\Delta \theta^{1,2} = \Delta \theta_{out}^{1,2} - \Delta \theta_{in}^{1,2}
\] (2.5)

Similarly relative phase difference between the eigenmodes is calculated using Eq. (2.6).

\[
\Delta \theta = \Delta \theta_{out}^{1} - \Delta \theta_{out}^{2}
\] (2.6)
2.2 Review of Previous Methods

Previously in the field of wavelength selection devices, the tunable LC FPI was proposed as an alternative to mechanical devices\textsuperscript{30,31,32}. Although LC FPI devices have compact non-mechanical structure and low power consumption, inherent polarization sensitivity is a big drawback. Although previous methods to reduce polarization sensitivity of LC FPI significantly differed from one another, the underlying principle was to reduce the phase difference between orthogonal eigenmodes traversing through the LC medium. For example, Patel\textsuperscript{33} proposed to use 90° twist LC FPI in the high field regime where half-way through the LC medium the distinct phase retardations experienced by two eigenmodes are switched, thus canceling the effective phase difference between these modes. Similarly Morita and others have\textsuperscript{34,35} suggested having two crossed quarter-wave retarders sandwiching a homogeneous LC layer (whose optic axis is 45° to theirs) within the FP cavity. In this configuration linear polarization of the eigenmodes gets rotated 90° passing through the quarter-wave retarder upon reflection from the mirrors resulting in equivalent path length for the two eigenmodes as they pass through the cavity twice. In these methods where the phase difference between the eigenmodes is minimized, the resonance peaks of the eigenmodes overlap and are simultaneously tuned as the LC is switched with external field. It is also possible to have polarization insensitive operation where resonance peaks of the eigenmodes do not overlap and only one of them is continuously tuned with external field. This is what Lee\textsuperscript{36} achieved with hybrid anchored LC FPI where the LC molecules are homeotropic on one
surface and axially homogeneous on the other. All these methods have advantages such as continuous tunability and high resolution which make them good candidates for wavelength division multiplexing applications.

Another approach to polarization insensitive operation is to have a very high twist rate, where mode mixing reduces the difference between effective indices of the eigenmodes. In effect when the twist rate increases such that the wavelength of light is much greater than the pitch of the helical structure, LC becomes optically isotropic. Although it is possible to switch high twist rate LC FPI by unwinding the helix with external field, switching threshold field is large and the resulting director deformation is not uniform.

### 2.3 New Device Concept

The polarization insensitivity requirement is satisfied in our approach not by reducing the phase difference between the eigenmodes but by simultaneously satisfying the resonance conditions of each eigenmode independently. At intermediate twist angles of LC (larger than waveguide regime\textsuperscript{37} but smaller than highly twisted state that is optically isotropic) the eigenmodes are generally elliptically polarized waves with ellipticity defined by parameters such as thickness, pitch and indices of the LC. In the rotating frame, polarization states of the eigenmodes do not change, because elliptically polarized eigenmodes follow the helical structure of the director such that the effective indices of individual eigenmodes stay constant. It was shown earlier that the phase
condition for a resonance peak in FPI is an integer multiple of half-wave. As in priori publications on the subject this condition is satisfied when phases of the eigenmodes are equal and are integer multiple of half-wave $m\pi$, $n\pi$ where $m = n$ where $m$ and $n$ are integers. But this condition is also satisfied even when phases of the eigenmodes are not equal as long as the individual phases of the eigenmodes are different integer multiples of half-wave $m\pi$, $n\pi$ where $m \neq n$. Since phase retardation of the eigenmodes increase with increasing twist angle it is possible to find a twist angle where individual phases of the eigenmodes fulfill integer multiple of half-wave requirement. If such twist angle where each eigenmode reaches integer multiple of half-wave ($m\pi$ and $n\pi$) simultaneously is found, then a polarization independent non-split resonance peak is expected.

However, we must also consider the effect of polarization change upon reflection from the FP mirrors. When an elliptically polarized eigenmode, $E_1$ travels through the FP cavity it picks up $m\pi$ phase, and upon reflection from one of the mirrors it will be decomposed into two eigenmodes, $E_1$ and $E_2$. This is due to the fact that unlike linearly polarized light when an elliptically polarized eigenmode reflects off the mirror its polarization state changes such that it is no longer a pure eigenmode but becomes a combination of pure eigenmodes. These two eigenmodes will pick up additional $m\pi$ and $n\pi$ phases as they travel back through the cavity making the total phases for $E_1$ and $E_2$ to be $2m\pi$ and $(m+n)\pi$ respectively. These waves will further be decomposed into separate eigenmodes as they reflect from the other mirror and will pick up additional phases. The same is true for the second eigenmode as it too will be decomposed into separate
eigenmodes upon reflection from the FP mirrors. Figure 2.2 illustrates how the eigenmodes are decomposed upon reflection from the mirrors and the total phase shift acquired as they travel through the cavity. From Figure 2.2 it can be seen that unless

\[ n = m + 2l \quad (l = 1, 2, 3, \ldots) \]

is true, reflected eigenmodes will be out of phase with each other which destroys the resonance condition. This means that the relative phase difference between the eigenmodes in the local frame must be an integer multiple of \(2\pi\) in order to satisfy the resonance condition.

In summary, the requirements for polarization insensitive operation of intermediate twist rate LC FPI are based on the resonance conditions of transmittance maximum and the interference conditions of reflected eigenmodes. The first condition is met when the phases of the eigenmodes are different integer multiples of half-wave while
the second condition is met when the relative phase difference between the eigenmodes reach integer multiple of full wave. For a given cavity thickness and LC material parameters, a twist angle that satisfies these requirements must be found.

2.4 Numerical Simulations and Results

As mentioned in the previous section, phases of the eigenmodes as they travel through the LC medium must be tuned to specific conditions. In this section we will introduce the numerical procedures to find the twist angle that satisfies polarization independent operation requirements. First, polarizations of the eigenmodes for given twist LC medium are found using Poincare sphere method and then Jones calculus\cite{38,39,40,41} for light propagation through birefringent media was used for phase calculations. Using these methods we will model hypothetical LC FPI where LC layer thickness is 0.45\(\mu\)m, and ordinary and extraordinary indices are 1.5 and 1.9 respectively. Dielectric mirrors were modeled as a stack of 6 alternating layers of TiO\(_2\) (n=2.27) and MgFl (n=1.38)\cite{42,43,44}. The optical thickness of each layer is tuned to be near \(\lambda/4\) thick at 550nm to get average reflectance of around 80\%. Transmittance spectrum of LC FPI for coherent normally incident light source was calculated using Berreman 4x4 matrix formulation\cite{45,46} which is a direct solution of Maxwell's equations in one dimension. In this method individual optical elements are represented by 4x4-matrix that depends on the dielectric and other optical parameters of the material. In our model LC FPI consists of 12 layers of isotropic dielectric material and 1000 layers of anisotropic birefringent
slabs each with thickness ~0.45nm representing the LC medium. Transmittance was normalized by the total unpolarized incident light and we ignored the effects of glass substrates, conducting and alignment layers, because they can be minimized and or compensated in real devices.

First, homogeneous LC FPI where the eigenmodes are linearly polarized lights parallel and perpendicular to the director is simulated. We modeled two separate LC FPI, one with polarizer along the director axis and the other perpendicular to the director axis and transmitted intensities are shown in Figure 2.3.

![Transmittance Vs Wavelength](image)

Figure 2.3: Transmittance of non-twist LC FPI (optical structure: \((n_L n_H)^3 n_{LC}(n_H n_L)^3\) where, \(n_L=1.38\), \(n_H=2.27\), \(n_{LC}=1.5/1.9\), \(d_L\sim95\text{nm}\), \(d_H\sim57\text{nm}\), \(d_{LC}=0.45\mu\text{m}\), twist angle=0)
In a highly twisted LC FPI, light sees an approximately isotropic medium with index equal to the average of ordinary and extraordinary indices. In this case there is a single polarization independent transmission peak at a wavelength where resonance condition is met (see Figure 2.4).

![Transmittance Vs Wavelength](image)

**Figure 2.4:** Transmittance of highly twisted LC FPI ($d_{LC}=0.45\mu m$, $n_{LC}=1.5/1.9$, twist angle=$3600^\circ$)

However, LC twist rate must be very high in order for the effective index to equal to the average of ordinary and extraordinary indices. Instead resonance condition for transmission maximum at that wavelength can be found for each eigenmode at intermediate twist angle. Once eigenmodes were determined using Poincare sphere, the twist LC structure is discretized ($N=1000$) and output electric vectors in the rotating
frame is calculated using Eq. (2.3). Relative phase difference between the eigenmodes in the rotating frame is calculated using equation 2.6 and plotted as function of twist angle in Figure 2.5.

Figure 2.5: Plot of relative phase difference between the eigenmodes as function of twist angle for 525nm wave (d_{LC}=0.45\mu m, n_{LC}=1.5/1.9). Note that phase difference reaches 2\pi at 169^\circ twist angle.

The acquired phase of each eigenmode as light travels through the LC cavity is calculated at the twist angle where phase difference between the eigenmodes reaches 2\pi. In the rotating frame output electric vectors from discrete birefringent layers can be calculated as \((\vec{E}_i^{1,2})_{out} = (M^i)^t(\vec{E}_i^{1,2})_{in}\) where \(i = 1, 2, 3...N\) and phase at thickness, \(x = (d / N)i\) is
calculated using Eq. (2.5). Phases of the eigenmodes in the local frame and transmitted intensity for 0.45\(\mu\)m cavity 169° twist LC FPI is shown in Figure 2.6.

Figure 2.6: Phases of the eigenmodes in the local frame and the transmitted intensity of twisted LC FPI (\(d_{LC}=0.45\mu\)m, \(n_{LC}=1.5/1.9\), \(\lambda_{\text{phase}}=525\text{nm}\), twist angle=169°)

In order to confirm our analysis we modeled LC FPI where phases of the eigenmodes did not satisfy the resonance conditions for polarization insensitive transmission peak. For example, left plot in Figure 2.6 shows that when the thickness is at around 0.23\(\mu\)m (twist angle\~0.23x169/0.45) phases of the eigenmodes reach \(l\pi\) and \(2\pi\) in the rotating frame. Although this satisfies the resonance condition for transmittance maximum for each eigenmode, interference condition is not met and as a result peak splitting occurs (see Figure 2.7).
Figure 2.7: Transmitted intensity of twisted LC FPI (d_{LC}=0.23\mu m, n_{LC}=1.5/1.9, twist angle=86^\circ)

Alternatively from Figure 2.5 the twist angle where relative phase difference between the eigenmodes reach 4\pi in the rotating frame can be found. At this twist angle phases of the eigenmodes in the local frame and the transmitted intensity are calculated and plotted in Figure 2.8.
Figure 2.8: Phases of the eigenmodes in the local frame and the transmitted intensity of twisted LC FPI ($d_{LC}=0.45\mu m$, $n_{LC}=1.5/1.9$, $\lambda_{phase}=525$nm, twist angle=340°)

We numerically calculated the LC director configuration as a function of applied voltage by minimizing the free energy which consists of Frank-Oseen elastic energy and electric energy. Material parameters of the modeled LC are: $n_e=1.9$, $n_o=1.5$, $\varepsilon_{ll}=21$, $\varepsilon_{\perp}=6$, $\gamma=0.083$PaS, $K_{11}=15$pN, $K_{22}=7$pN, $K_{33}=30$pN. Here, $K_{11}$ to $K_{33}$ are elastic constants, and $\gamma$ is the rotational viscosity of the LC material. We assumed infinite anchoring energy such that LC molecules on substrate surfaces are fixed permanently with 2° pretilt angle. Detailed numerical method for calculating the director configuration of twisted LC structure can be found in Berreman's paper on LC twist cell dynamics. Figure 2.9 shows what happens when external field is applied across the FP cavity and twist LC structure is unwound. As the twist is unwound transmittance peak splits and moves toward shorter wavelength region before finally reaching single peak at around 480nm.
Figure 2.9: Transmitted intensity of twist LC FPI as function of applied voltage 
($d_{LC}=0.45\mu m$, $n_{LC}=1.5/1.9$, twist angle=169°)

2.5 Experimental Verifications

2.5.1 Numerical Calculation

The experimental verification of the LC etalon device in this chapter serves two 
purposes of which the main purpose is to demonstrate polarization independent 
transmission and the secondary purpose is to probe viability of pico-projector light 
modulator based on the LC etalon. We will not cover many details regarding the 
modulator but will only study its limitations from the polarization independent operation 
point of view. More detailed presentation on transmissive LC etalon based pico-
projection device will be the subject of Chapter 4 of this dissertation.
For the demo we have designed highly reflective dielectric mirror using titanium dioxide and magnesium fluoride stack. Figure 2.10 shows the reflectance of the dielectric mirrors. Using a pair of this mirror we built tunable twisted LC etalon demo device.

![Figure 2.10: Reflectance spectrum of dielectric mirror](image)

In the process of building the sub-micron thickness LC etalon demo device we had to:

i) Complete a process development for sub-micron cell assembly

ii) Control twist angle and switching in sub-micron cell

iii) Determine the effects of thickness non-uniformity, light incident angle, and additional optical layers (conducting, alignment, substrate, etc) on transmittance
The etalon mirrors were coated with additional thin film coating on top of the dielectric stack for LC alignment purposes. The final optical structure of the mirror is $n_G n_{ITO}(n_H n_L)^4 n_{Hn_A}$ where G, ITO, H, L, and A indices stand for glass substrate, conducting layer, high and low indices of dielectric materials, and alignment layer.

The etalon LC material is a commercial material, BL009, from Merck with indices, $n_H \sim 1.81$ and $n_L \sim 1.53$ at 550nm wavelength.

At first, we will ignore the effects of glass substrates, conducting and alignment layers in the optical calculations. Figure 2.11 shows the transmittance spectrum of a 3rd order device as in Figure 2.9 only this time with high reflectivity dielectric mirrors, and lower birefringence LC material. In the figure there is a high transmittance in 450nm region, because of the leaky dielectric mirror. Tunability is reduced as a result of lower birefringence LC material. Regardless of these shortcomings we can still demonstrate polarization independent switching in the visible region using this design. We expect the etalon to be transmissive for cyan (495nm) and red (620nm) region in the unwound state, and green (520nm) region in the twist state.
Simulation result shows that cyan (495nm) and red (620nm) transmission peaks are not quite 100% even though the cell is in a polarization insensitive homeotropic state. This is due to a small polarization splitting as a result of residual birefringence near the surfaces of the etalon. Etalon thickness and LC twist angles are chosen so that the green (520nm) transmission peak is polarization independent as shown in Figure 2.11. The width of green transmission peak is only about 1nm which means that even slightest change in the thickness or in the twist angle would result in polarization splitting. Sensitivity to these variables will allow us to analyze the requirements of polarization independent transmission.

We would like to see the effect of thickness variations on the performance of the device. Figure 2.12 and Figure 2.13 show what happens when the target thickness is...
missed by 30nm. In both cases not only the transmission peaks have shifted significantly but also polarization peak splitting occurred. 30nm variation caused almost 30% drop in the green transmission peak amplitude. It should be noted that for the demo, due to the delicate process development of thin cells we are likely to miss the target thickness by as much as 30nm or more. One of the factors influencing this variation is the alignment layer which will put on additional 50nm on top of the spacing posts. The thickness of the spacing posts can be controlled, however, depending on the cell construction process alignment layer on top of these posts might be destroyed. Also further deviations in the thickness can occur if the LC material gets on top of these posts during the filling process.

Figure 2.12: Modeled transmittance of 3rd order etalon device ($d_{LC}=0.45\mu m$ (solid)/$0.48\mu m$ (dashed), $n_c=1.81$, $n_o=1.53$, $\phi=163^\circ$)
Figure 2.13: Modeled transmittance of 3rd order etalon device \(d_{\text{LC}}=0.45\mu\text{m}\) (solid)/0.42\mu\text{m} (dashed), \(n_e=1.81\), \(n_o=1.53\), \(\phi=163^\circ\)

Similarly a change in incident angle of the light source would also cause polarization splitting for the green transmission peak. However, we expect this effect to be much less pronounced due to the low order of the LC etalon. Indeed Figure 2.14 shows very slight change in transmission peaks for off-axis \((\pm 10^\circ)\) light.
Figure 2.14: Modeled transmittance of 3rd order etalon device (\(d_{\text{LC}}=0.45\mu\text{m}, n_e=1.81, n_o=1.53, \phi=163^\circ\), incident angle=0\(^\circ\) (solid)/±10\(^\circ\) (dashed))

We would also like to see the tolerance of the device towards the change in the twist angle of LC. We know that polarization independent transmission at 520nm is achieved when the twist angle is \(~163^\circ\). Figure 2.15 shows the transmittance plot of the device when the twist angle deviates from 163\(^\circ\) by ±5\(^\circ\). It can be seen that when the twist angle is varied polarization splitting caused the peak transmittance to drop from \(~85\%\) down to \(~55\%\).
Figure 2.15: Modeled transmittance of 3rd order etalon device (d_{LC}=0.45\mu m, n_e=1.81, n_o=1.53, \phi=163^\circ/163^\circ\pm5^\circ \text{(bold)}, \text{incident angle}=0^\circ)

Finally, the effects of glass substrates and conducting layers on the transmittance is calculated and shown in Figure 2.16. As one would expect there is about 8\% reduction in the transmittance due to front and back reflections from the glass substrates. Other than that glass substrates and conducting layers have no major effect on the performance of the device.
Figure 2.16: Modeled transmittance showing effects of glass substrates and conducting layers

In conclusion, although numerical calculations predict that the demo device would not be sufficient as a projection device demo, because of the low light transmittance and contrast, it will be able to demonstrate polarization independent tuning of transmission peaks in the visible range.

2.5.2 Thin Cell Fabrication Process

Without considering the fabrication of uniform LC cell with thickness under 1µm, development of spacer technology alone is a difficult task. Previously, we have tried several methods involving thin films where post spacers were formed with the help of patterned photolithographic process. Thin films that were tried but were unsuccessful
were polymer with an epoxy cross linking resist, and inorganic SiO$_2$. In most cases films
cracked in the process or yielded non-uniform post spacers that resulted in very thick cell
gaps.

Subsequently we developed a process for sputter coating of SiO$_2$ post spacers on
the substrates using shadow masks. A hexagonal close packed grid of circular holes with
30µm diameter and 300µm pitch was formed by a photolithographic process on 14µm
thin aluminum foil. These masks were then applied to cell substrates and were put in a
vacuum chamber for SiO$_2$ sputter coating. Figure 2.17 shows the optical microscopy
image of the post spacers. Due to the high temperature and high pressure of sputter
coating, the mean free path of the coating particles are very short compared to the
diameter of the holes and as a result lot of the material coated the side walls. In addition,
problems that arose during the process were the lifting off of masks from the substrates
during the vacuum pumping process and difficulties in separating the aluminum mask
without damaging the post spacers. Optical microscopy studies of the substrates revealed
that the post spacers were non-uniform and in some places completely washed out. Final
assembly of cells using these substrates yielded very non-uniform thicknesses.
Two main problems that resulted in the failure of sputter coating process were non-uniform adhesion of the aluminum mask to the plate surfaces, and the short mean free path of the particles due to high pressure and high temperature. To solve these problems we coated 1.5-2µm thick photoresist on the plates and patterned the same hexagonal close packed grid of holes on them. Essentially the photoresist acted as a shadow mask only this time with no problems with the adhesion to the plates. Next, instead of sputter coating SiO$_2$ we decided to do evaporative coating of SiO$_x$. Usually this material is used as an alignment layer, and is not as robust compared with the SiO$_2$. However, evaporative coating is not high pressure and high temperature as sputter coating and as a result there would not be much building up of material on the side walls of the hole. At the end of the coating we dissolved the photoresist in acetone using ultrasonic bath. The main problems associated with the sputter coating process were solved, and indeed we were able to make clean uniform spacer posts (see Figure 2.18).
We have constructed 2 test cells using normal glass substrates with SiO\textsubscript{x} post spacers. Test cells were held together inside a vacuum bag without any adhesives and the thickness was calculated from the fit of reflection curve from the spectrometer data. Figure 2.19 and Figure 2.20 show the result of fitting of the two test cells. Test cells were relatively uniform in most places except few areas where thickness increases due to particle contamination. However, when pressed down with mechanical force, thickness becomes uniform in both cells, measuring ~240nm in one and ~290nm in the other. The measured thickness of SiO\textsubscript{x} post spacers during the evaporation process is around 250nm. This means that the process yielded uniformity within 15% of the target thickness in this range. This result may get better as the thickness of the coating increases. At
present, the LC etalon demo device requires thickness within range of 450nm with tolerance less than 10% (<±45nm).

Figure 2.19: Test cell 1 (thicker spot on the left and thinner spot on the right)

Figure 2.20: Test cell 2 (thicker spot on the left and thinner spot on the right)
Once a post spacer process was developed and successfully tested on cells with normal glass substrates, we proceeded toward making the demo device. The thickness requirement for the demo is 0.45µm which is thicker than the test cells that we constructed. We coated the dielectric mirror substrates with 0.45µm thick SiO$_x$ post spacers and assembled two cells. During the assembly process a thin line of UV curable glue was applied to one of the substrates and the other substrate was put on top before placing the cell under a vacuum mat. Under the vacuum mat cells were physically pressed until glues are well dispersed and a uniform thickness area in the middle of the cell is found. At this point cell was put under UV light for ~5min so that the glue is completely cured. Figure 2.21 and Figure 2.22 shows the transmittance of empty cell consisting of pair of dielectric mirror coated substrates. It can be seen that transmission peak does not move significantly when the spectrometer is moved 1mm from the center position. This means that the thickness of the etalon cavity varies only a few nm across the 2mm region in the center of the cell, which is more than adequate for the demo devices.
Figure 2.21: Transmittance of Non-Filled Etalon Cell

Figure 2.22: Transmittance of Non-Filled Etalon Cell

Figure 2.23 shows the comparison between experiment and the modeling where the fitted thickness is around 0.36\(\mu\)m. During the evaporative coating process crystal quartz
measured the thickness of the posts to be around 0.45µm. However, when we measured the thickness of the spacing layer using ellipsometer it was around 0.38µm. This means that the measurement of crystal quartz needs to be calibrated with the actual thickness for further coatings.

![Transmittance Vs Wavelength](image)

Figure 2.23: Comparison between experimental data and modeled data

### 2.5.3 LC Filling Process

Next we filled the etalon cell with LC material with chiral agent. The chiral concentration was calculated such that for a given alignment layers of the mirrors LC material would have twist angle around 160°. LC cell was filled in vacuum chamber and was heated to ~110°C on a hot plate. During the heating and cooling process LC material goes through the Isotropic-Nematic transition which would help us in getting uniform
alignment. Figure 2.24 shows the LC texture image. It can be seen that there are lots of air bubbles got inside the cell during the heating and cooling process. In addition, there is non-uniformity in the texture revealed by slight color changes between dark blue and pinkish region. We think the color change is due to different twist states which are caused by variations in thickness or anchoring condition or both.

![Figure 2.24: Optical Polarizing Microscopy image of LC texture](image)

Indeed, when the thickness was measured again around the region where there is no LC material present, it was expanded from ~0.36µm to 0.65µm (see Figure 2.25).
Figure 2.25: Transmittance near the entrance region where there is no LC (thickness expanded from 0.36µm to 0.65µm)

For the next cell, we decided not to go through the heating and cooling process and decided to immediately seal the cell before LC material completely fills it. In this way we would prevent thermal expansion and contraction, and would have help from capillary forces of the material to hold the substrates together tightly. The thickness of this cell was measured before the filling process and was determined to be ~0.36µm which is again in good agreement with the thickness of the SiOx spacing layer. Figure 2.26 shows the LC texture of the new cell. Except for the dark stripe patterns LC texture is a uniform dark blue in most regions indicating good alignment even without the Isotropic-Nematic transition. One concern is that LC material crawled up on top of some of the spacers which can be seen by color change in the spacer when rotated between crossed polarizers.
This may eventually cause thickness non-uniformity when substrates get lifted off the spacers.

![Figure 2.26: Optical Polarizing Microscopy image of LC texture](image)

We would also like to get rid of the stripe patterns as they may cause unwanted spectrum variations. Figure 2.27 shows the non-voltage applied transmittance spectrum of the filled LC etalon cell as compared to modeled data. It can be seen that the model and the experiment agrees well in terms of the location of the peaks, however, polarization splitting is more severe in the experimental data. Such polarization splitting can be caused by a variation in the twist angle which can also explain the stripe patterns in the LC texture. Indeed, when we modeled the LC etalon with twist angle of $20^\circ$ instead of $160^\circ$, experimental result agrees much better with the model result (See Figure 2.28).
From here we conclude that the stripe pattern is the region where LC twist angle is 160° and is the cause of small transmission peak between the splitted peaks at around 525nm.

Figure 2.27: Comparison between modeled (φ=163°) and experimental data for filled LC etalon cell
Unfortunately, before we got rid of the stripe patterns in the cell, thickness of the cell got changed over 1 day period. LC texture shows that air bubbles got inside the cell (the dark region) and different twist states (red and blue regions) formed which is a result of variations in the thickness uniformity (see Figure 2.29).
2.6 Summary

If the FP cavity thickness and LC parameters are known, then polarization insensitive twist angle can be determined from the plot of relative phase difference between the eigenmodes in the rotating frame (see Figure 2.5). At this twist angle phases of the eigenmodes in the rotating frame reach $\sim 4\pi$ and $\sim 2\pi$ (see left plot in Figure 2.6) which satisfies the resonance conditions for polarization independent transmittance peak. 

Corresponding effective indices in the rotating frame are calculated using 

\[
\left(n_{\text{eff}}\right)_{1,2} = \phi_{1,2}/2\pi d, \quad \text{where} \quad \phi_{1,2} = 4\pi \text{ and } 2\pi \quad \text{and are found to be } 2.28 \text{ and } 1.11 \quad \text{respectively. Note that the effective indices can also be calculated analytically as}
\]

\[
\left(n_{\perp}\right)^2 = \epsilon_{\parallel,\perp} + \alpha^2 \left(1 \pm 2\pi / \delta\right)
\]

where, $\epsilon_{\parallel,\perp}$ are dielectric permittivities along and perpendicular to the LC director, $\alpha = \lambda / p$, $\bar{\epsilon} = (\epsilon_{\parallel} + \epsilon_{\perp}) / 2$, and $\delta = (\epsilon_{\parallel} - \epsilon_{\perp}) / 2$. Results of analytical calculation agree well with that of the numerical solution up to 2nd decimal point. Analytical equation for effective indices can also be used to calculate the phase difference between the eigenmodes for twist LC structure and get the same result as shown in Figure 2.5. Note that even before effective indices of the eigenmodes reach the average index of the LC, the resonance condition is met and transmittance spectrum looks the same as that of isotropic FPI with average index of the LC (see right plot in Figure 2.6 and Figure 2.4).

Figure 2.7 shows transmittance plots of two cases where peak splitting in resonance peaks indicate polarization sensitivity. In this FPI phases of the eigenmodes reach $1\pi$ and $2\pi$ in the rotating frame (from left plot in Figure 2.6). However, this
condition does not satisfy the polarization insensitive resonance peak requirements and as a result large peak splitting occurs. Alternatively, a twist angle at which phases of the eigenmodes reach $\sim 1\pi$ and $\sim 5\pi$ (left plot in Figure 2.8) can be found from Figure 2.5 where polarization insensitive transmittance peak reappears (right plot in Figure 2.8).

Figure 2.9 shows tunability of LC FPI as external field is applied across the cavity. As the LC twist is unwound resonance conditions break down and transmission peak splits. We note here that although peak splitting occurs, the transmission peak does not completely separate into two peaks, and instead, when the field strength is high enough so that director aligns perpendicular to the cavity surface, peak splitting disappears. This would not be the case in a high finesse etalon where the transmission peaks are sharper and the separation of adjacent peaks is higher. In such case there will be complete peak splitting of the eigenmodes in the intermediate voltage range. The same explanation applies to the order of the peak where in the case of Figure 2.9 low order peak which has wider transmission bandwidth has less splitting as compared to the higher order peak.

Possible applications of such device are tunable optical filters and polarization independent light modulator in display systems. For example, the switch shown in Figure 2.9 can be used as modulator for field sequential color system where three light sources, one for each of the R, G, and B wavelength regions, illuminate the modulator in a time sequential manner. Depending on the wavelength of the illuminated light the modulator either transmits or blocks the light. In such wide transmission bandwidth applications
polarization peak splitting due to slight variations in the twist angle would not hurt the performance, because the distance between the split peaks are much shorter than the transmission bandwidth.

Generally LC etalons in optical filters are usually considered as high order narrow bandwidth transmittance filters whose transmittance is not very high and as a result they are not suited for the application mentioned above. For example, in Patel's 90° twist cell significant tunability requires high order etalon as a result of field-on operation. Similarly additional retarders within the LC cavity as suggested by Morita also increases the etalon order. In the cases where the resonance peaks of the eigenmodes do not overlap as in Lee's method, the tunable light throughput is same as that of polarization dependent modulator which is not acceptable for devices with high light transmission requirement.

In conclusion, we have determined the requirements for polarization independent LC FPI for intermediate twist LC structure. These are,

1. The phases of each eigenmode in the local frame must be integer multiple of $\pi$
2. The relative phase difference between the eigenmodes in the local frame must be integer multiple of $2\pi$

The first requirement comes from the resonance condition of transmission peak in FPI, whereas the second requirement comes from the interference condition between the eigenmodes as they reflect from the FP mirrors.

High reflectivity dielectric mirrors were fabricated and thin cell assembly process was developed for experimental verification of the polarization independent LC etalon.
Although missing the target thickness and imperfect processing resulted in polarization splitting in the experimental LC etalon transmittance, model predictions agree well with the experimental results. Better demonstration of polarization independent LC etalon tunability will be presented in the Chapter 4.

2.7 Appendix 2A

2.7.1 Jones Matrix Method

Jones matrix method for light propagation was first introduced by R. C. Jones in 1941. Applications where determining the effects of birefringent material on the polarization state of light and where interference effects are neglected are ideal for using Jones Matrix method. In this method plane waves with given polarization state is expressed in terms of Jones vectors (complex) and various optical elements such as birefringent retarders are expressed by 2X2 Jones matrices. More specifically incident and transmitted light through birefringent medium is related by 2x2 matrix in the following way.

\[
\bar{J}_{out} = J J_{in}
\]  

(2A-1)

In general Jones vector for plane wave is of the form \( \bar{J} = \begin{pmatrix} A_x e^{i\delta_x} \\ A_y e^{i\delta_y} \end{pmatrix} \) where the electric field is the real component of the complex variable such that \( E_x(t) = \text{Re}(A_x e^{i(\omega t + \delta)}) \) and \( E_y(t) = \text{Re}(A_y e^{i(\omega t + \delta)}) \). If the amplitude of the plane wave is not important in the calculation
than the normalized Jones vectors are used where \( \mathbf{j} \cdot \mathbf{j} = 1 \). For example, X or Y plane polarized light can be represented as \( \mathbf{j} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) or \( \mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \) and right or left circular polarized light can be represented as \( \mathbf{j} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \) or \( \mathbf{j} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \). An optical slab of birefringent media with birefringence \( \Delta n \) can be expressed in the matrix form as:

\[
M = \begin{bmatrix}
\exp(i\Gamma_e) & 0 \\
0 & \exp(i\Gamma_o)
\end{bmatrix}
\]

(2A-2)

Where, \( \Gamma_{e,o} = 2\pi n_{e,o}d / (\lambda) \) with \( n_{e,o} \) and \( d \) being the extra-ordinary, ordinary indices of the birefringent medium and the thickness of the medium respectively. For example, such matrix can represent liquid crystal medium which has optical anisotropy represented by ordinary and extraordinary indices which are indices perpendicular and parallel to the long axis of the average molecular orientation called the director. It is also possible to represent twisted liquid crystal structure where the director traces helix with axis perpendicular to the cell normal with Jones matrix. If the total twist angle of the LC is \( \phi \), LC structure is divided into infinitesimally small \( N \) number of birefringent slabs each placed in \( \phi / N \) angle with respect to its neighboring slab. Then the matrix becomes \( \Gamma_{e,o} = 2\pi n_{e,o}d / (N\lambda) \) and rotation matrix \( R(\phi / N) \) is used to transform the electric field vector into correct frame of reference.

\[
R(\phi) = \begin{bmatrix}
\cos (\phi / N) & \sin (\phi / N) \\
-\sin (\phi / N) & \cos (\phi / N)
\end{bmatrix}
\]

(2A-3)
The matrix \( M' = MR(\varphi / N) \) and input and output electric vectors in the rotating frame are related as:

\[
(\vec{E}')^{1,2}_{\text{out}} = (M')^N \vec{E}^{1,2}_n
\]  

(2A-4)

Electric vectors can be rewritten in the lab frame by multiplying with a rotation matrix as:

\[
(\vec{E})^{1,2}_{\text{out}} = R(\varphi)(\vec{E}')^{1,2}_{\text{out}}
\]  

(2A-5)

Using the above equations we can calculated the phase, \( \theta \), of the propagating electric vectors by rewriting the electric vectors in the form \( \vec{E} = (E_x \exp(i\theta), E_y) \).

**Copy of MATLAB Codes**

**Program 1 - Phase Calculation Program**

```matlab
%% This program calculates phases of the eigenmodes as light wave propagates through twisted media.

function Phase

clear all;
cclc;
imag=sqrt(-1);
lambda=.525;  \% wavelength of light in micrometers
phi=0*pi/180;  \% twist angle
d=0.5;  \% LC thickness in micrometers
ne=1.78;  \% LC extraordinary index
no=1.53;  \% LC ordinary index
dn=ne-no;  \% LC birefringence
```

```matlab
%% This program calculates phases of the eigenmodes as light wave propagates through twisted media.

function Phase

clear all;
cclc;
imag=sqrt(-1);
lambda=.525;  \% wavelength of light in micrometers
phi=0*pi/180;  \% twist angle
d=0.5;  \% LC thickness in micrometers
ne=1.78;  \% LC extraordinary index
no=1.53;  \% LC ordinary index
dn=ne-no;  \% LC birefringence
```
gammae = 2*pi*ne*d/lambda;
gammao = 2*pi*no*d/lambda;
gamma = gammae - gammao;
u = dn*d/lambda;
dpitch = phi*0.5/pi;

omega = atan(2*dpitch/u)/2;  
% find the elevation angle of cone in the Poincare sphere. Note that
% tan(omega) = -b/a is the ellipticity of the eigenmodes

a = sqrt(1./(tan(omega).^2+1));  
% find long and short axis' of the ellipse
b = tan(omega).*a;
Ei1 = [a; -imag*b];  
% Eigenvectors (see pg105 in "Optics of LC Displays")
Ei2 = -imag*[b; imag*a];

N = 1000;  
% number of discrete layers

M0 = [exp(-imag*gammae/N)*cos(phi/N), exp(-imag*gammae/N)*sin(phi/N);
    -exp(-imag*gammao/N)*sin(phi/N),
    exp(-imag*gammao/N)*cos(phi/N)];
% Jones matrix of individual layers in rotating frame

phase1(1:N) = 0;
phase2(1:N) = 0;
Ef1 = Ei1;
Ef2 = Ei2;

for i=1:N
    Ef1 = M0*Ef1;  
    % calculate new field in rotating frame after passing through a single layer
    Ef2 = M0*Ef2;
    phase1(i) = angle(Ef1(1))-angle(Ei1(1));  
    % calculate phase accumulated as the wave propagates through
    % the LC cell
    phase2(i) = angle(Ef2(2))-angle(Ei2(2));
end

phase_diff = phase1-phase2;
x(1:N) = (0:d/(N-1):d);
figure; clf;
Program 1 - Verification

Eigenmodes in homogeneous non-twist LC medium are ordinary and extraordinary modes with indices $n_o$ and $n_e$ respectively. Figure 2A.1 below shows the plot of phases of the eigenmodes as they propagate through the birefringent medium. Here, LC cell thickness is 0.5µm and ordinary and extraordinary indices are 1.53 and 1.78. Then the phases can be calculated simply by $2\pi n_{e,o}d/\lambda$ and the results are $2.91\pi$ and $3.39\pi$ respectively. These results agree well with the numerical calculation results.
Program 2 - Phase Difference of the Eigenmodes

%%% This program calculates phase difference between eigenmodes as light wave propagates through twisted media and plots it as function of twist angle

function Phase_Difference

clear all;
clc;

imag=sqrt(-1);

lambda=.525;  \% wavelength in micrometers

d=0.38; \% LC cell thickness in micrometers

ne=2.13; \% LC extraordinary index

no=1.55; \% LC ordinary index

dn=ne-no; \% LC birefringence

gammae=2*pi*ne*d/lambda;
gammao=2*pi*no*d/lambda;
u=dn*d/lambda;

N=500;

phi_i=0; \% initial twist angle

phi_f=360*pi/180; \% final twist angle

dphi=(phi_f-phi_i)/(N-1);

phi(1:N)=(phi_i:dphi:phi_f);

dpitch(1:N)=phi(1:N)*0.5/pi;

omega(1:N)=atan(2*dpitch(1:N)/u)/2; \% find the elevation angle of cone in Poincare sphere. Note that % tan(omega)=-b/a is the ellipticity of the eigenmodes

a(1:N)=sqrt(1./(tan(omega(1:N)).^2+1));
b(1:N)=tan(omega(1:N)).*a(1:N); \% find long and short axis' of the ellipse
M0=eye(2);

phase_diff(1:N)=0;

N2=1000;

for i=1:N
    Ei1=[a(i); -imag*b(i)]; %see pg105 in "Optics of LC Displays"
    Ei2=-imag*[b(i); imag*a(i)];
    M0=[exp(-imag*gammae/N2)*cos(phi(i)/N2),
        exp(-imag*gammae/N2)*sin(phi(i)/N2);
        -exp(-imag*gammao/N2)*sin(phi(i)/N2),
        exp(-imag*gammao/N2)*cos(phi(i)/N2)]; %Jones matrix of individual layers in rotating frame
    Ef1=Ei1;
    Ef2=Ei2;

    for j=1:N2
        Ef1=M0*Ef1;
        Ef2=M0*Ef2; %calculate new field in rotating frame after passing through a single layer
    end

    phase_diff(i)=(angle(Ef1(1))-angle(Ei1(1))) -
                   (angle(Ef2(2))-angle(Ei2(2))); %difference between phases accumulated by eigenmodes
end

figure; clf;

line(phi*180/pi, phase_diff/pi, 'Color', 'b', 'LineWidth', 2, 'LineStyle', '-');

grid on;

xlabel('Twist Angle (deg)');
ylabel('Phase shift');
title('Phase Shift Vs Twist Angle');

Program 2 - Verification
Figure below shows the plot of relative phase difference of the eigenmodes in the rotating frame as function of LC twist angle. Here, LC cell thickness is 0.5\(\mu\)m and ordinary and extraordinary indices are 1.53 and 1.78. Phase difference can be calculated for non-twist medium by \(\Gamma = 2\pi \Delta n d / \lambda\) which is \(\sim 1.50\) which agrees well with the numerical calculation.

**Fig 2A.2:** Phases of the eigenmodes (\(\lambda = 0.525\mu\)m, \(d = 0.5\mu\)m, \(n_e = 1.78\), \(n_o = 1.53\))

2.7.2 Berreman 4X4 Matrix Method (Prof. Peter Palffy-Muhoray's textbook on Liquid Crystal Materials was used for the derivations of the method here\(^49\))

\[
\vec{E} = \vec{E}_0 e^{i(\omega r - k \cdot r)}
\]

\[
\vec{E} = \vec{E}_0 e^{-ik_z z} e^{i(\omega r - k \cdot r)} = \vec{E}'(z) e^{i(\omega r - k_z z)}
\]

Likewise magnetic field vector can be written as, \(\vec{H} = \vec{H}'(z) e^{i(\omega r - k_z z)}\)
\[ \nabla \times \bar{E} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \left\{ \begin{array}{c} i \left( -ik_y E_y' - \frac{\partial E_x'}{\partial z} \right) - j \left( -ik_z E_z' - \frac{\partial E_y'}{\partial z} \right) + \\
+ k \left( -ik_x E_x' + ik_y E_y' \right) \right\} e^{i\omega(k_x x, k_y y, k_z z)} = -i \omega \mu \bar{H} \]

\[ \nabla \times \bar{H} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \left\{ \begin{array}{c} i \left( -ik_y H_y' - \frac{\partial H_x'}{\partial z} \right) - j \left( -ik_z H_z' - \frac{\partial H_y'}{\partial z} \right) + \\
+ k \left( -ik_x H_x' + ik_y H_y' \right) \right\} e^{i\omega(k_x x, k_y y, k_z z)} = i \omega \varepsilon \bar{E} \]

We can assume non-magnetic material, \( \mu = \mu_0 \), and make the fields dimensionless by \( e_x = E_x' / E_0, \ h_x = H_x' / H_0 \) where \( H_0 = E_0 / Z_0 \) (\( Z_0 \) - impedance of free space). Also the distance can be made dimensionless by \( z' = k_0 z \) where, \( k_0 = 2 \pi / \lambda_0 \). Then,

\[-ik_y e_z - \partial e_y / \partial z' = -ih_x \]

\[ik_y e_z - \partial e_x / \partial z' = -ih_y \]

\[-ik_x e_y + ik_y e_z = -ih_z \]

Note \( k' = k / k_0 \) and similarly dividing the magnetic field equations by \( H_0 \) and \( k_0 \) gives,

\[-ik_y h_z - \partial h_y / \partial z' = i \left( \varepsilon_x e_x + \varepsilon_y e_y + \varepsilon_z e_z \right) \]

\[ik_y h_z - \partial h_x / \partial z' = i \left( \varepsilon_y e_x + \varepsilon_y e_y + \varepsilon_z e_z \right) \]

\[-ik_x h_y + ik_y h_z = i \left( \varepsilon_x e_x + \varepsilon_y e_y + \varepsilon_z e_z \right) \]
Where, $\varepsilon'_0 = \varepsilon_{yy} / \varepsilon_{a}$. Eliminating $h_z$ and $e_z$ by using

$$h_z = -k_y'e_x + k_x'e_y$$

$$e_z = -\varepsilon'_{xz} e_x / \varepsilon'_{zz} - \varepsilon'_{yz} e_y / \varepsilon'_{zz} + k_y' h_x / \varepsilon'_{zz} - k_x' h_y / \varepsilon'_{zz}$$

As a result,

$$e_z = \frac{\partial}{\partial z'} \begin{bmatrix} e_x \\
 e_y \\
 h_x \\
 h_y \end{bmatrix} = iA \begin{bmatrix} e_x \\
 e_y \\
 h_x \\
 h_y \end{bmatrix}$$

Where

$$\hat{A} = \begin{bmatrix}
\varepsilon'_{xk} k_x' / \varepsilon'_{zz} & \varepsilon'_{yk} k_y' / \varepsilon'_{zz} & -k_y' k_x' / \varepsilon'_{zz} & k_x'^2 / \varepsilon'_{zz} - 1 \\
\varepsilon'_{yk} k_y' / \varepsilon'_{zz} & \varepsilon'_{y} k_y' / \varepsilon'_{zz} & -k_y'^2 / \varepsilon'_{zz} + 1 & k_y' k_y' / \varepsilon'_{zz} \\
(k_y'^2 + \varepsilon_{yy}' - \frac{\varepsilon_{yx}' \varepsilon_y'}{\varepsilon'_{zz}}) & (-k_y'^2 + \varepsilon_{yy}' - \frac{\varepsilon_{yx}' \varepsilon_y'}{\varepsilon'_{zz}}) & \varepsilon_{yy}' k_y'^2 / \varepsilon'_{zz} & -\varepsilon_{yy}' k_y' / \varepsilon'_{zz} \\
(-k_y'^2 + \varepsilon_{yy}' + \frac{\varepsilon_{yx}' \varepsilon_y'}{\varepsilon'_{zz}}) & (-k_y'^2 + \varepsilon_{yy}' + \frac{\varepsilon_{yx}' \varepsilon_y'}{\varepsilon'_{zz}}) & -\varepsilon_{yy}' k_y' / \varepsilon'_{zz} & \varepsilon_{yy}' k_y'^2 / \varepsilon'_{zz} \\
\end{bmatrix}$$

or

$$\frac{\partial \hat{X}}{\partial z'} = i\hat{A} \hat{X}$$

Dividing the sample into thin slabs we can write

$$\hat{X}_{n+1} = \hat{X}_n + i\hat{A}_n \hat{X}_n \Delta z = \left( I + i\hat{A}_n \Delta z \right) \hat{X}_n$$
\[ \hat{X}_{n+1} = (\hat{I} + i\hat{A}_n \Delta z)(\hat{I} + i\hat{A}_{n-1} \Delta z)(\hat{I} + i\hat{A}_{n-2} \Delta z) \ldots (\hat{I} + i\hat{A}_1 \Delta z) \hat{X}_1 \]

\[ \hat{X}_{\text{out}} = \hat{b}\hat{X}_{\text{in}} \]

\[ \hat{b} = (\hat{I} + i\hat{A}_n \Delta z)(\hat{I} + i\hat{A}_{n-1} \Delta z)(\hat{I} + i\hat{A}_{n-2} \Delta z) \ldots (\hat{I} + i\hat{A}_1 \Delta z) \]

\[
\begin{bmatrix}
    e_x \\
    e_y \\
    h_x \\
    h_y_{\text{out}}
\end{bmatrix}
= \begin{bmatrix}
    b_{11} & b_{12} \\
    b_{21} & b_{22} \\
    b_{31} & b_{32} \\
    b_{41} & b_{42}
\end{bmatrix}
\begin{bmatrix}
    e_x \\
    e_y \\
    h_x \\
    h_y_{\text{in}}
\end{bmatrix}
\]

\[ \hat{X}_{\text{in}} = \hat{X}_i + \hat{X}_r \]

Where, \( X_i \) and \( X_r \) are vectors of incident and reflected fields. Then,

\[ \hat{X}_i = \hat{b}\hat{X}_i + \hat{b}\hat{X}_r \]

\[
\begin{bmatrix}
    e_{xx} \\
    e_{xy} \\
    h_{xx} \\
    h_{xy}
\end{bmatrix}
= \begin{bmatrix}
    b_{11} & b_{12} \\
    b_{21} & b_{22} \\
    b_{31} & b_{32} \\
    b_{41} & b_{42}
\end{bmatrix}
\begin{bmatrix}
    e_{xx} \\
    e_{xy} \\
    h_{xx} \\
    h_{xy}
\end{bmatrix}
+ \begin{bmatrix}
    b_{11} & b_{12} \\
    b_{21} & b_{22} \\
    b_{31} & b_{32} \\
    b_{41} & b_{42}
\end{bmatrix}
\begin{bmatrix}
    e_{rx} \\
    e_{ry} \\
    h_{rx} \\
    h_{ry}
\end{bmatrix}
\]

In the case of known incident light,

\[ \tilde{k}_i = \text{given} \]

\[ \tilde{k}_r = \tilde{k}_i - 2(\tilde{k}_i \cdot \hat{N}) \hat{N} \]

\[ \tilde{k}_i = \tilde{k}_i \]
If the sample is surrounded by same isotropic medium and if the wave vectors are known, then magnetic field components for the incident, reflected, and transmitted waves can be written in terms of the electric fields.

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]

\[ \tilde{k} \times \vec{E} = \omega \mu \tilde{H} \quad \text{since} \quad \tilde{h} = \tilde{H}Z_0 / E_0 \]

\[ \tilde{k} \times \vec{e} = \frac{\omega \mu}{kZ_0} \tilde{h} = \eta \tilde{h} \quad \text{where} \quad \eta = Z / Z_0 \]

Similarly, we can write \( \tilde{k} \times \tilde{h} = \tilde{e} / \eta \)

Combining these allows us to solve for \( h_x \) and \( h_y \) in terms of \( e_x \) and \( e_y \).

\[
\begin{bmatrix}
  e_x \\
  e_y \\
  h_x \\
  h_y \\
\end{bmatrix} =
\begin{bmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  -k_x k_y / \eta k_z & -(1-k_x^2)/\eta k_z & 0 & 0 \\
  (1-k_y^2)/\eta k_z & k_x k_y / \eta k_z & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
  e_x \\
  e_y \\
  h_x \\
  h_y \\
\end{bmatrix}
\]

For incident light we write,

\[
\begin{bmatrix}
  e_{ix} \\
  e_{iy} \\
  h_{ix} \\
  h_{iy} \\
\end{bmatrix} =
\begin{bmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  -k_{ix} k_{iy} / \eta_{ix} k_{iz} & -(1-k_{ix}^2)/\eta_{ix} k_{iz} & 0 & 0 \\
  (1-k_{iy}^2)/\eta_{iy} k_{iz} & k_{ix} k_{iy} / \eta_{iy} k_{iz} & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
  e_{ix} \\
  e_{iy} \\
  h_{ix} \\
  h_{iy} \\
\end{bmatrix}
\]

or
\[
\begin{bmatrix}
    e_{tx} \\
    e_{ty} \\
    h_{tx} \\
    h_{ty}
\end{bmatrix} = \hat{\alpha} \begin{bmatrix}
    e_{ix} \\
    e_{iy} \\
    0 \\
    0
\end{bmatrix}
\]

Similarly for transmitted light,

\[
\begin{bmatrix}
    e_{tx} \\
    e_{ty} \\
    h_{tx} \\
    h_{ty}
\end{bmatrix} = \begin{bmatrix}
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    -k_{tx}k_{ty} / \eta_t k_{tz} & - \left(1 - k_{tx}^2 \right) / \eta_t k_{tz} & 0 & 0 \\
    (1 - k_{ty}^2) / \eta_t k_{tz} & k_{tx} k_{ty} / \eta_t k_{tz} & 0 & 0
\end{bmatrix} \begin{bmatrix}
    e_{tx} \\
    e_{ty} \\
    0 \\
    0
\end{bmatrix}
\]

and for reflected light,

\[
\begin{bmatrix}
    e_{rx} \\
    e_{ry} \\
    h_{rx} \\
    h_{ry}
\end{bmatrix} = \hat{\beta} \begin{bmatrix}
    e_{tx} \\
    e_{ty} \\
    0 \\
    0
\end{bmatrix}
\]

\[
\begin{bmatrix}
    e_{rx} \\
    e_{ry} \\
    h_{rx} \\
    h_{ry}
\end{bmatrix} = \hat{\gamma} \begin{bmatrix}
    0 \\
    0 \\
    e_{tx} \\
    e_{ty}
\end{bmatrix}
\]

In terms of these, we get

\[
\hat{X}_i = b\hat{X}_i + b\hat{X}_r
\]
\[
\begin{bmatrix}
e_{rx} \\
e_{ry} \\
h_{rx} \\
h_{ry}
\end{bmatrix} = \hat{b}
\begin{bmatrix}
e_{ix} \\
e_{iy} \\
h_{ix} \\
h_{iy}
\end{bmatrix} + \hat{b}
\begin{bmatrix}
e_{rx} \\
e_{ry} \\
h_{rx} \\
h_{ry}
\end{bmatrix}
\]

and eliminating h gives:

\[
\hat{\beta}
\begin{bmatrix}
e_{rx} \\
e_{ry} \\
0 \\
0
\end{bmatrix} = \hat{b}\hat{\alpha}
\begin{bmatrix}
e_{ix} \\
e_{iy} \\
0 \\
0
\end{bmatrix} + \hat{b}\hat{\gamma}
\begin{bmatrix}
0 \\
0 \\
e_{rx} \\
e_{ry}
\end{bmatrix}
\]

Now noting that

\[
\hat{\beta}
\begin{bmatrix}
e_{rx} \\
e_{ry} \\
0 \\
0
\end{bmatrix} = \tilde{\beta}
\begin{bmatrix}
e_{rx} \\
e_{ry} \\
e_{rx} \\
e_{ry}
\end{bmatrix}
\]

and

\[
\hat{\gamma}
\begin{bmatrix}
0 \\
0 \\
e_{rx} \\
e_{ry}
\end{bmatrix} = \tilde{\gamma}
\begin{bmatrix}
e_{rx} \\
e_{ry} \\
e_{rx} \\
e_{ry}
\end{bmatrix}
\]

we get

\[
(\hat{\beta} - \tilde{\beta}\hat{\gamma})
\begin{bmatrix}
e_{rx} \\
e_{ry} \\
e_{rx} \\
e_{ry}
\end{bmatrix} = \hat{b}\hat{\alpha}
\begin{bmatrix}
e_{ix} \\
e_{iy} \\
0 \\
0
\end{bmatrix}
\]
and

\[
\begin{pmatrix}
e_{rx} \\
e_{ry} \\
e_{rx} \\
e_{ry}
\end{pmatrix}
= \left( \hat{\beta} - \hat{\gamma} \right)^{-1} \hat{b}\hat{\alpha}
\begin{pmatrix}
e_{ix} \\
e_{iy} \\
0 \\
0
\end{pmatrix}
\]

Copy of MATLAB Codes

Program 3 - Transmission Calculations in Layered Media

%%% Berreman 4x4 Matrix Method for Wave Propagation

function Berreman

clc;
clear all;

imag=sqrt(-1);
N=[0,0,1]; % slab normal
theta_i=0; % incident angle
I=eye(3);
ne=1.9; % extraordinary refractive index
no=1.5; % ordinary refractive index
thickness=.45e-6; % LC thickness in meters

path1 = 'C:\MATLAB701\work\Top_Mirror.dat'; % input data
path2 = 'C:\MATLAB701\work\Bottom_Mirror.dat'; % input data
path3 = 'C:\MATLAB701\work\dir_config10V.dat'; % input data
TM=load(path1);
BM=load(path2);
Dir=load(path3);

[top_layers,dump] = size(TM);
[bot_layers,dump] = size(BM);
[dir_layers,dump] = size(Dir);

num_layers=top_layers+bot_layers+dir_layers;

num=200;

lambda_i=450e-9;
lambda_f=650e-9;

dlambda=(lambda_f-lambda_i)/(num-1);

lambda(1:num)=(lambda_i:dlambda:lambda_f);

r_intensity(1:num)=0;
t_intensity(1:num)=0;

dLC=thickness/dir_layers;

d_tot(1:num_layers)=0;

d_tot(1:top_layers)=TM(1:top_layers,1);

d_tot(top_layers+1:top_layers+dir_layers)=dLC;

d_tot(top_layers+dir_layers+1:num_layers)=BM(1:bot_layers,1);

n_o(1:num_layers)=0;

n_o(1:top_layers)=TM(1:top_layers,2)-imag*TM(1:top_layers,3);

n_o(top_layers+1:top_layers+dir_layers)=no;

n_o(top_layers+dir_layers+1:num_layers)=BM(1:bot_layers,2)-imag*BM(1:bot_layers,3);

n_e(1:num_layers)=n_o(1:num_layers);

n_e(top_layers+1:top_layers+dir_layers)=ne;

theta(1:num_layers)=pi/2;

theta(top_layers+1:top_layers+dir_layers)=pi/2-Dir(1:dir_layers,1);

phi(1:num_layers)=1e-9;

phi(top_layers+1:top_layers+dir_layers)=Dir(1:dir_layers,2);

ni=1.0;    % incident medium index
nt=1.0;  %exit medium index
dz=0;
b=eye(4);
A=eye(4);
[evec,eval]=eig(A);
alpha=eye(4);
beta=eye(4);
gamma=eye(4);
epsr=eye(3);
ki=[0, 0, 0]; kt=ki; kr=ki;
field_out1=[0, 0, 0, 0];
field_out2=[0, 0, 0, 0];
for i=1:num
    ki=[0, sin(theta_i), cos(theta_i)]*2*pi/lambda(i);  %incident k vector
    kt=ki;  %transmitted k vector
    kr=ki-2*(ki*N')*N;  %reflected k vector
    kt=kt/norm(ki);  %non-dimensionalize kt, kr, and ki
    kr=kr/norm(ki);
    ki=ki/norm(ki);
    alpha=[1, 0, 0, 0; 0, 1, 0, 0; -ni*ki(1)*ki(2)/ki(3), -ni*(1-ki(1)^2)/ki(3), 0, 0; ni*(1-ki(2)^2)/ki(3), ni*ki(1)*ki(2)/ki(3), 0, 0];
    beta=[1, 0, 0, 0; 0, 1, 0, 0; -nt*kt(1)*kt(2)/kt(3), -nt*(1-kt(1)^2)/kt(3), 0, 0; nt*(1-kt(2)^2)/kt(3), nt*kt(1)*kt(2)/kt(3), 0, 0];
    gamma=[0, 0, 1, 0; 0, 0, 0, 1; 0, 0, -ni*kr(1)*kr(2)/kr(3), -ni*(1-kr(1)^2)/kr(3); 0, 0, ni*(1-kr(2)^2)/kr(3), ni*kr(1)*kr(2)/kr(3)];
b = eye(4);

for j = 1:num_layers
    dz = d_tot(j)*2*pi/lambda(i);
    epsr(1,1) = n_o(j)^2 + (n_e(j)^2 - n_o(j)^2)*cos(theta(j))^2*cos(phi(j))^2;
    epsr(1,2) = (n_e(j)^2 - n_o(j)^2)*cos(theta(j))^2*sin(phi(j))*cos(phi(j));
    epsr(1,3) = (n_e(j)^2 - n_o(j)^2)*sin(theta(j))*cos(theta(j))*cos(phi(j));
    epsr(2,1) = epsr(1,2);
    epsr(2,2) = n_o(j)^2 + (n_e(j)^2 - n_o(j)^2)*cos(theta(j))^2*sin(phi(j))^2;
    epsr(2,3) = (n_e(j)^2 - n_o(j)^2)*sin(theta(j))*cos(theta(j))*sin(phi(j));
    epsr(3,1) = epsr(1,3);
    epsr(3,2) = epsr(2,3);
    epsr(3,3) = n_o(j)^2 + (n_e(j)^2 - n_o(j)^2)*sin(theta(j))^2;

    %% Differential Propagation Matrix (A-Matrix)
    A = eye(4);
    A(1,1) = epsr(3,1)*ki(1)/epsr(3,3);
    A(1,2) = epsr(3,2)*ki(1)/epsr(3,3);
    A(1,3) = -ki(1)*ki(2)/epsr(3,3);
    A(1,4) = ki(1)^2/epsr(3,3)-1;
    A(2,1) = epsr(3,1)*ki(2)/epsr(3,3);
    A(2,2) = epsr(3,2)*ki(2)/epsr(3,3);
    A(2,3) = -ki(2)^2/epsr(3,3)+1;
    A(2,4) = ki(1)*ki(2)/epsr(3,3);
    A(3,1) = ki(1)*ki(2)+epsr(2,1)-epsr(2,3)*epsr(3,1)/epsr(3,3);
    A(3,2) = -ki(1)^2+epsr(2,2)-epsr(3,2)^2/epsr(3,3);
    A(3,3) = epsr(2,3)*ki(2)/epsr(3,3);
    A(3,4) = -epsr(2,3)*ki(1)/epsr(3,3);
    A(4,1) = ki(2)^2-epsr(1,1)+epsr(3,1)^2/epsr(3,3);
    A(4,2) = -ki(1)*ki(2)-epsr(1,2)+epsr(3,1)*epsr(3,2)/epsr(3,3);
A(4,3)=-epsr(1,3)*ki(2)/epsr(3,3);
A(4,4)=epsr(1,3)*ki(1)/epsr(3,3);
[evect,eval]=eig(A);
b=evect*expm(imag*eval*dz)*inv(evect)*b;
end  %end of for

field_out1=inv(beta-b*gamma)*b*alpha*[1;0;0;0];
field_out2=inv(beta-b*gamma)*b*alpha*[0;1;0;0];
r_intensity(i)=(abs(field_out1(3))^2+abs(field_out1(4))^2+abs(field_out2(3))^2+abs(field_out2(4))^2)/2;
t_intensity(i)=(abs(field_out1(1))^2+abs(field_out1(2))^2+abs(field_out2(1))^2+abs(field_out2(2))^2)/2;
end  %end of for

line(lambda*1e9, t_intensity, 'Color', 'c','LineWidth',2,'LineStyle','-');
axis([lambda(1)*1e9 lambda(num)*1e9 0 1]);
grid on;

Program 3 - Verification

This program needs input files which are top and bottom mirror files and the director configuration file which consists of theta and phi angles of the director within the cavity. Figure below show sample input files where three columns of data for the mirror file represent thickness, real, and imaginary indices whereas two columns of data for the director file represent theta and phi angles of the director. It should be noted here that the LC layer is divided into \( N \) number of slabs and for each slab there is corresponding theta and phi director angles.
For verification purposes we calculated the transmittance of LC etalon that was modeled in Figure 2.3 of Chapter 2. Input files are the same as shown above where the director angles are constant throughout the LC layer (θ=88°, φ=0°). The result is consistent with the analytical calculation of Figure 2.3 in Chapter 2.
2.7.3 Liquid Crystal Director Simulation - Theta/Phi Method

The liquid crystal director configuration as a function of applied voltage is calculated using well known theta-phi method. Although the theta-phi method is known for instability in the high field regime, and is limited to 1D structure only, it is easy to implement and the results are sufficient for our calculations. Following figure shows the configuration used in the numerical program.

Figure 2A.4: LC etalon transmittance (input files are shown in figure A3)
Figure 2A.5: Coordinate system used in the theta-phi method for director calculation

Here, theta is the angle from the normal to the cell and phi is the angle from the x-axis.

Starting with the Frank-Oseen free energy equation\textsuperscript{50, 51}:

\[ f = \frac{1}{2} K_{11} (\nabla \cdot \hat{n})^2 + \frac{1}{2} K_{22} (\hat{n} \cdot \nabla \times \hat{n} - q_0)^2 + \frac{1}{2} K_{33} (\hat{n} \times \nabla \times \hat{n})^2 - \frac{1}{2} D \cdot E \]

We can write it using the cylindrical coordinate system as:

\[
\begin{align*}
    f &= \frac{1}{2} \left\{ K_i \sin^2 \theta \left( \frac{\partial \theta}{\partial z} \right)^2 + K_j \left[ \sin^4 \theta \left( \frac{\partial \phi}{\partial z} \right)^2 + 2 q \sin^2 \theta \left( \frac{\partial \phi}{\partial z} \right) + q^2 \right] + K_k \left[ \cos^2 \theta \left( \frac{\partial \theta}{\partial z} \right)^2 + \sin^2 \theta \cos^2 \theta \left( \frac{\partial \phi}{\partial z} \right)^2 \right] \right\} \\
    &\quad + \frac{D^2}{2 \varepsilon_0 (\varepsilon_0 \cos^2 \theta + \varepsilon_1 \sin^2 \theta)}
\end{align*}
\]

The equilibrium director configuration is found by minimizing the free energy,\[ F = \int f dz \], using Euler Lagrange equations:

\[
\left( \frac{\partial f}{\partial \theta} - \frac{d}{dz} \frac{\partial f}{\partial z} \right) = 0 \quad \text{and} \quad \left( \frac{\partial f}{\partial \phi} - \frac{d}{dz} \frac{\partial f}{\partial z} \right) = 0
\]
This problem can be solved numerically by dividing the cell into \( N \) layers, and for each layer calculating \( \theta \) and \( \phi \) that satisfies the Euler Lagrange equations. When the fluid inertia is negligible the dynamics can be obtained with the torque balance equations:

\[
\frac{\partial f}{\partial \theta} - \frac{d}{dz} \frac{\partial f}{\partial \theta'} = -\gamma \frac{\partial \theta}{\partial t}
\]

\[
\frac{\partial f}{\partial \phi} - \frac{d}{dz} \frac{\partial f}{\partial \phi'} = -\gamma \sin^2 \theta \frac{\partial \phi}{\partial t}
\]

Above equations can be discretized using an infinitesimal time step \( \Delta t \):

\[
\theta_{\text{new}} = \theta_{\text{old}} + \Delta \theta
\]

\[
\Delta \theta = -\frac{\Delta t}{\gamma} \left[ \frac{\partial f}{\partial \theta} - \frac{d}{dz} \frac{\partial f}{\partial \theta'} \right]
\]

\[
\phi_{\text{new}} = \phi_{\text{old}} + \Delta \phi
\]

\[
\Delta \phi = -\frac{\Delta t}{\gamma} \left[ \frac{\partial f}{\partial \phi} - \frac{d}{dz} \frac{\partial f}{\partial \phi'} \right]
\]

At each discrete grid point the Euler Lagrange equations for \( \theta \) and \( \phi \) are evaluated, and new values of \( \theta \) and \( \phi \) are obtained. These new values are stored in a temporary array until we have scanned through the whole system. At this point the older array of \( \theta \) and \( \phi \) variables is overwritten and the process continues until the difference in the new and old values reaches a certain tolerance. At that point the system is considered to be in an equilibrium configuration. It can be easily seen that the above equations cause instability when the angle theta approaches 0 because of undefined \( \phi \).

In this program we consider a fixed potential so that at each loop through the system we reset the z component of the displacement vector, \( \Delta z \), to a value set by the constant potential requirement. This is done as follows:
Since \( E = V/d \) and \( D = \varepsilon_0 \vec{E} \) \( \Rightarrow \) \( D = \frac{V}{d} \varepsilon_0 \vec{E} \).

Here, \( \frac{1}{\varepsilon} = \left( \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_3} + \ldots \right) / N \) and \( \varepsilon_i = (\varepsilon_{||} \cos^2 \theta_i + \varepsilon_\perp \sin^2 \theta_i) \)

Therefore, for a constant potential:

\[
D_\varepsilon = \frac{V\varepsilon_0 N}{d \sum_i \frac{1}{\varepsilon_i}}
\]

Copy of MATLAB Codes

Program 4 - Liquid Crystal Director Calculation - Theta-Phi Method

```matlab
%% DYNAMICS METHOD FOR CONSTANT POTENTIAL

function Director_Calculation

format long

voltage=0;
e0 = 8.8542;
dtG=0.000001; %time constant
N = 40; %number of layers
d=5.0; %cell thickness in micrometer
dz=d/(N-1); %increment step
k1 = 6.65;
k2 = 3;
k3 = 8.95;
epara = 19.5;
```
eperp = 8;
delta_e = epara-eperp;
d_p=0;
q=2*pi/(d/d_p);
theta(1:N) = 88*pi/180; %initialize theta
twist_angle = 90*pi/180;
phi(1:N) = -twist_angle/2+(0:(N-1))*dz*twist_angle/d; %initialize phi
theta_n(1:N) = theta(1:N);
phi_n(1:N) = phi(1:N);
z(1:N) = (0:N-1)*dz;
dF = -1;
while ((dF < -1E-15))
    for i=2:(N-1)
        dtdz = 0.5*(theta(i+1)-theta(i-1))/dz;
        dtdz2 = (theta(i+1)+theta(i-1)-2*theta(i))/dz^2;
        dpdz = 0.5*(phi(i+1)-phi(i-1))/dz;
        dpdz2 = (phi(i+1)+phi(i-1)-2*phi(i))/dz^2;
        ezz = eperp+delta_e*cos(theta(i))^2;
        sum_ezz = 0;
        for j=1:N
            sum_ezz = sum_ezz + d/(eperp+delta_e*cos(theta(j))^2)/N;
        end
        dtheta=0.5*dtdz^2*sin(2*theta(i))*(k3-k1) + ... 
        dpdz^2*sin(2*theta(i))*(k2*sin(theta(i))^2+0.5*k3*cos(2*theta(i)))+...
        dpdz*k2*q*sin(2*theta(i)) + ...
        dtdz2*(-k1*sin(theta(i))^2-k3*cos(theta(i))^2)+...
        0.5*voltage^2*delta_e*e0*sin(2*theta(i))/(ezz*sum_ezz)^2;
        dphi=dpdz2*(k2*sin(theta(i))^4+0.25*k3*sin(2*theta(i))^2)+...
\[
dp dz \ast \ast dt dz \ast \sin(2 \ast theta(i)) \ast \ast (2 \ast k2 \ast \sin(theta(i))^2 + k3 \ast \cos(2 \ast theta(i))) + \ldots \\
dt dz \ast (k2 \ast q \ast \sin(2 \ast theta(i))); \\
theta_n(i) = theta(i) - dtG \ast dtheta; \\
phi_n(i) = phi(i) + dtG / \sin(theta(i))^2 \ast dphi; \\
end
sum_{ezz} = 0; \\
sum_{ezz \_n} = 0; \\
for j=1:N \\
\sum_{ezz} = \sum_{ezz} + d / (eperp + delta_e \ast \cos(theta(j))^2) / N; \\
\sum_{ezz \_n} = \sum_{ezz \_n} + d / (eperp + delta_e \ast \cos(theta_n(j))^2) / N; \\
end
Dz = voltage \ast e0 / sum_{ezz}; \\
Dz_n = voltage \ast e0 / sum_{ezz \_n}; \\
F_{init} = 0; \\
F_{final} = 0; \\
for i=2:(N-1) \\
dt dz = 0.5 \ast (theta(i+1) - theta(i-1)) / dz; \\
dp dz = 0.5 \ast (phi(i+1) - phi(i-1)) / dz; \\
dt dz_n = 0.5 \ast (theta_n(i+1) - theta_n(i-1)) / dz; \\
\dp dz_n = 0.5 \ast (\phi_n(i+1) - phi_n(i-1)) / dz; \\
F_{init} = F_{init} + (0.5 \ast k1 \ast \dt dz^2 \ast \sin(theta(i))^2 + 0.5 \ast k2 \ast (dp dz^2 \ast \sin(theta(i))^4 + 2 \ast q \ast dp dz \ast \sin(theta(i))^2 + q^2) + 0.5 \ast k3 \ast \cos(theta(i))^2 \ast (\dt dz^2 + dp dz^2 \ast \sin(theta(i))^2) + 0.5 \ast Dz^2 / e0 / (eperp + delta_e \ast \cos(theta(i))^2)) \ast dz \\
0.5 \ast k3 \ast \cos(theta(i))^2 \ast (dt dz^2 + dp dz^2 \ast \sin(theta(i))^2) + 0.5 \ast Dz^2 / e0 / (eperp + delta_e \ast \cos(theta(i))^2)) \ast dz; \\
F_{final} = F_{final} + (0.5 \ast k1 \ast dt dz_n^2 \ast \sin(theta_n(i))^2 + 0.5 \ast k2 \ast (dp dz_n^2 \ast \sin(theta_n(i))^4 + 2 \ast q \ast dp dz_n \ast \sin(theta_n(i))^2 + q^2) + 0.5 \ast k3 \ast \cos(theta_n(i))^2 \ast (dt dz_n^2 + dp dz_n^2 \ast \sin(theta_n(i))^2) + 0.5 \ast Dz^2 / e0 / (eperp + delta_e \ast \cos(theta_n(i))^2)) \ast dz; \\
0.5 \ast k3 \ast \cos(theta_n(i))^2 \ast (dt dz_n^2 + dp dz_n^2 \ast \sin(theta_n(i))^2) + 0.5 \ast Dz^2 / e0 / (eperp + delta_e \ast \cos(theta_n(i))^2)) \ast dz;
\]
end

F_init=F_init;
F_final=F_final;
dF=F_final - F_init;
if (dF < 1e-25)
    dtG= dtG*1.005;
    theta(1:N) = theta_n(1:N);
    phi(1:N) = phi_n(1:N);
else
    dtG = dtG/1.05;
    dF = -1;
end

end

thetaphi(1:N,1:2)=0;
thetaphi(1:N, 1)=theta;thetaphi(1:N, 2)=phi;
save dir_config.dat thetaphi -ascii;
line(z, 90-theta*180/pi, 'Color', 'k','LineWidth',2, 'LineStyle', '-');
ax1 = gca;
title('Theta/Phi Vs Thickness');
xlabel('z (um)');
ylabel('Theta (deg)');
ylabel('Phi (deg)');
ax2 = axes('Position',get(ax1,'Position'), 'XAxisLocation','bot','YAxisLocation','right', 'Color','none', 'XColor','k','YColor','k');
line(z, phi*180/pi, 'Color', 'b', 'LineWidth', 2, 'LineStyle', '--', 'Parent', ax2);
ylabel('Phi (deg)');

Program 4 - Verification
Director calculation of simple nematic TN cell as function of applied voltage is shown in figure below for verification. Here, liquid crystal material parameters are that of 5CB ($K_{11}=6.65\text{pN}$, $K_{22}=3\text{pN}$, $K_{33}=8.95\text{pN}$, $\varepsilon_{//}=19.5$, $\varepsilon_{\perp}=8$, $d=5\ \mu\text{m}$).

![Figure 2A.6: LC Director Angles as Function of Applied Voltage](image)

Figure 2A.6: LC Director Angles as Function of Applied Voltage
Chapter 3

Transmissive Liquid Crystal Etalon Device

3.1 Introduction

3.1.1 Function and Requirements of the Device

An optical device that could modulate multiple wavelength regions, with near 100% transmission in the “open” state, would be of high value for many applications. For example, a field sequential light modulator that has the appropriate characteristics could significantly advance a new class of projection displays sometimes called “pico-projectors”\textsuperscript{52,53,54}. The characteristics needed for this highly portable device are very high transmission to maximize the image brightness, reasonable contrast, and compatibility with field sequential color generation. In a field sequential color system, red (R), green (G), and blue (B) light sources illuminate the modulator in a time sequential manner so that at any instant of time light valve either transmits or blocks the wavelength region corresponding to the light source that is on. In order for the modulator to operate a field sequential color system above the flicker perception rate of the eye, each illumination interval must be less than about 5 milliseconds. This leads to a modulator switching time on the order of one millisecond. However field sequential color systems running at this rate are known to suffer from “image break-up”\textsuperscript{55}, so higher field rates, and faster switching times of the modulator are desired.
LC devices have been previously considered for projection applications\textsuperscript{8}. The most common approach to light modulation is to change the polarization state of light which requires the use of polarizers that lower the light transmission to less than 50\%. Additional polarization recycling components result in limited device mobility and lower cost efficiency. On the contrary light scattering and diffractive devices that are polarization insensitive have been considered as well. However, the overall optical efficiency of the projection optics with these devices has not offered a significant improvement because of the narrow cone of light that can be collected in the transmissive state\textsuperscript{56}.

This chapter covers the design and simulation results of transmissive liquid crystal etalon device. The main goal of the chapter is to envision a new type of high efficiency low power mobile display based on a liquid crystal etalon device.

Previously, LC etalon devices have been used in tunable optical filters such as wavelength division multiplexing (WDM) systems in telecommunications\textsuperscript{30}. However, difficulty in achieving high finesse, considerable loss and limited tunability associated with LC etalons limit their widespread usage\textsuperscript{57}. We propose two different LC etalon designs, a polarization dependent light modulator for monochromatic laser light sources and a polarization independent modulator for broad band light sources. We show polarization insensitive operation using a new method previously introduced by the authors in which phase difference between the elliptically polarized eigenmodes in twist LC structure is tuned to be an integer multiple of $2\pi$\textsuperscript{58}. Our approach to polarization independent operation fits field sequential light modulator applications, offering high
transmittance over broad wavelength region. As mentioned in Chapter 1 of the dissertation current active matrix displays with absorbing polarizers and side-by-side color filters are very power inefficient and as a result new technology is required for future mobile display applications. The main aspects related to the design of LC etalon for transmissive field sequential display application are:

1) Transmission

► Current polarization based field sequential devices can achieve high contrast over the entire visible spectrum. The main problem associated with transmission for these devices is polarizer loss which reduces the overall efficiency down to around 40%. Therefore, acceptable transmission to justify the need for new device is significantly more than 40%.

► Efficient transmission of light requires the ability to modulate the spectrum of light that backlight unit generates. For commercial LED backlights this could mean modulating spectra as broad as 35nm.

► Single cell design of field sequential display requires the same device to be able to modulate such spectra for all three primary colors.

2) Contrast

► For large screen applications, contrast of the device needs to be more than 300. However, for mobile display applications good contrast may be about 100.
Issues related to increasing the contrast for LC etalon-based device are connected in large part to the topics covered in transmission.

3) Switching Time

Current developments in field sequential designs show that field time less than 5ms is required for acceptable refresh rate. This means LC response time must be less than 1ms.

High switching speed of thin etalon devices need to be exploited. If switching time around 0.1ms can be achieved for these devices, then there will be clear advantage over polarization based designs in terms of speed.

4) Voltage Requirement

Existing active matrix displays work with 6 volts, and ability of the new technology to adapt the same backplanes requires the same voltage range

5) Integration Into the Display Systems

Feasibility for projection display applications

Viewing angle for direct view display applications

6) Manufacturability

Spacer design

Cell assembly and filling procedures for ultra thin LC cavity
This list is in descending order in terms of the importance of each aspect covered in this chapter. For instance, high efficiency transmission is the foremost design requirement and therefore will be the main subject of this chapter whereas integration into the display systems and manufacturability are only slightly covered in this chapter and in the chapter on demo device.

3.1.2 Overview of the Physics

Figure 3.1 shows the basic structure of parallel plate etalon device. Here, etalon plates are coated with reflecting surfaces (dielectric, metal, or hybrid) and are separated with spacers. Incident light undergoes multiple reflections within the etalon cavity and total transmittance of the etalon is determined by several parameters shown in the right side of Figure 3.1.

Figure 3.1: Parallel plate etalon device
Here, + and - signs signify the direction of reflectance/transmittance from the interface between adjacent mediums. For instance, reflection from the front surface, \( r_a^+ \), is not necessarily equal to the reflection from the inner surface, \( r_a^- \), of that same interface if there is absorption in the layers.

The underlying equation for the transmittance of etalon device is shown below. Note that the amplitude coefficients of Figure 3.1 (in small letters) are converted to the intensity coefficients (in capital letters) in the equation.

\[
T = \frac{T_a T_b}{\left(1 - \sqrt{R_a R_b}\right)^2} \left[1 + \frac{4\sqrt{R_a R_b}}{\left(1 - \sqrt{R_a R_b}\right)^2} \sin^2 \left(\frac{\phi_a + \phi_b - \delta}{2}\right)\right] \quad \text{Eq. (3.1)}
\]

Here, \( T_{a,b} \), \( R_{a,b} \) are transmittance and reflectance of etalon mirrors, \( \phi_{a,b} \) is the phase change on reflection from the mirrors, and \( \delta = \frac{2\pi n_{eff} d}{\lambda} \) where, \( n_{eff} \) and \( d \) are effective index and thickness of the mirror cavity. This equation reveals that the transmittance of the etalon can be tuned by several parameters including, thickness, \( d \), refractive index, \( n \), and phase factors, \( \phi_a \) and \( \phi_b \).

It's helpful to analyze the transmittance of etalon devices with simplified equation where phase changes upon reflection are ignored, and where two mirrors are identical. Although in real devices phase change on reflection and absorption (especially in metal mirrors) cause large deviation in transmittance, we can approximate what happens in real devices with good accuracy if we know the underlying principles. With these conditions,
transmittance, $T$, of the etalon is maximum for $\delta=m\pi$, where $m=1, 2, 3...$, and minimum halfway between these values. We will assume that the thickness, $d$, is fixed and the only variable by which the transmittance is tuned is refractive index, $n$, of the cavity. LC etalon that modulates polarized light source will be optimized before polarization independent design is considered.

The human eye is most sensitive to the green region of the visible spectrum and luminous efficiency drops sharply as wavelength diverges from this region$^{59}$. Figure 3.2 shows the luminous efficiency as function of wavelength of light. Etalon tunability of at least 200nm, from 450nm to 650nm, is required in order to generate full color image.

![Figure 3.2: Luminous efficiency of human eyes](image)
If we were to construct 1st order \((m=1)\) etalon device that can tune over this region of visible spectrum, then a 0.16\(\mu\)m cavity etalon device filled with LC with birefringence, \(\Delta n\), of 0.5 is required (see Figure 3.3). Although it is advantageous to use 1st order device because of wider transmittance peaks, there are several problems that need to be solved. For instance, in such a low-order cavity, LC will not be able to switch at low voltages (especially if the LC is in the twist configuration). In addition, as we will see later, phase changes associated with real mirrors would further reduce the thickness of the device in order to modulate 1st order transmittance peaks. When we increase the order of the device to 2nd order, the width of transmittance peaks is considerably narrowed (see the 2nd order device in Figure 3.4). If broad band light sources instead of laser are to be considered as backlight, then the narrowing of peaks will decrease the effective brightness of the display. The width of transmittance peaks are controlled by the order of the device and by the reflectivity of mirrors. Increased reflectivity reduces transmission, therefore resulting in narrow peaks, whereas decreased reflectivity broadens transmittance peaks.
Figure 3.3: Transmittance curve of 1st order LC etalon device w/ 0.9 reflective mirrors

Figure 3.4: Transmittance curve of 2nd order LC etalon device w/ 0.9 reflective mirrors
Figure 3.5 shows the transmittance of 2nd order device with reduced reflectivity mirrors. Here, transmittance peaks are almost identical to those in Figure 3.3, except that minimums in the transmission are slightly elevated. This shows that decreased reflectivity of mirrors would increase the light leakage in the dark state at the same time widening the transmission peaks in the bright state. As a result, going any higher than 2nd order would further reduce the width of transmittance peaks and decrease in mirror reflectivities would not be able to compensate them without considerably dropping the contrast of the device.
Figure 3.6: Relative intensity Vs wavelength (commercial Luxeon K2 LEDs from Philips)

Figure 3.6 shows the relative intensity distribution of commercial LEDs. Here, spectrum of blue and red LEDs are considerably narrow with FWHM < 30nm, whereas that for green spectrum is around 35nm. The 2nd order device in Figure 3.5 has FWHM ~ 25nm and would be able to transmit most of the light if the polarization dependency is reduced.

3.1.3 Dielectric Mirror Phase Shift on Reflection and Phase Dispersion Effects

Now, with clear ideas on the limitations of mirror reflectivities and the order of the device we can analyze devices with real mirrors where phase shift and absorption are not negligible. Detailed information on the phase shift on reflection from the dielectric mirrors and the phase dispersion effects are covered in the appendix 3A and this section
Dielectric mirrors are made of a $1/4\lambda$ stack of alternating high and low index dielectric materials. Mirror reflectivities are easily tuned by the total number of layers in a stack and by the ratio of high and low indices. Although increasing the number of alternating layers in general increases the mirror reflectivity, depending on the mismatch of indices at the entrance and exit medium reflectivity can decrease even with increasing number of layers. In the same way overall reflectivity of the mirror can be enhanced by increasing the ratio of high and low indices of the dielectric materials.

If dielectric layers of a mirror stack are $1/4\lambda$ thick, then phase shift on reflection for the center wavelength is 0, or $\pi$ depending on the index of the dielectric layers relative to the surrounding medium. For example, at the center wavelength a mirror with high index material next to air interface results in $\pi$ phase shift, whereas low index material results in 0 phase shift. However, in most cases phase shift is not exactly equal to but is close to 0 or $\pi$, because of the frequency dispersion effect. As it will be shown in the following sections this phase change on reflection causes the etalon transmission peaks to shift to longer wavelength regions. In addition, phase change dispersion reduces half width of these peaks. We already know that half width of peaks gets wider when transmittance of etalon gets bigger (decreased reflectivity mirrors). It’s also known that transmittance increases and decreases as function of wavelength because of the phase factor within the sine function (see Eq. (3.1)). When phase change on reflection from the
mirrors enters the sine function, rate of change of transmittance relative to the wavelength gets larger. This is a direct result of dispersion of phase changes on reflection. Faster variation of transmittance of the etalon relative to the wavelength causes the transmittance peaks to be sharper. This can be described by simple rise in the slope of transmittance relative to the wavelength ($dT/d\lambda$).

To illustrate these effects we modeled low order dielectric mirrored etalon device. If we assemble 0.16µm etalon device using 90% reflective dielectric mirrors, we expect to see transmittance curve similar to that in Figure 3.3. However, when dielectric mirrors are used transmittance peaks are shifted and the tunability is significantly reduced (see Figure 3.7). This discrepancy is a result of phase change on reflection from the mirror surfaces.

![Transmittance Vs Wavelength](image)

Figure 3.7: Transmittance curve of LC etalon device w/ 0.9 reflective dielectric mirrors
The phase shift on reflection from dielectric mirrors causes transmission peaks to shift to longer wavelength regions causing an apparent decrease in the order of the device. This shift in peak wavelengths is especially severe for low order devices making the design of low order etalon device complicated. In addition, the half width of the transmission peak is noticeably reduced in Figure 3.7 compared to those in Figure 3.3. The decrease in half width can be calculated using Eq. (3A.11) in Appendix 3A, \[
\frac{(n_H-n_L)}{(n_H-n_L+n_L/m)},
\]
where \(n_H=2.27\), \(n_L=1.38\), and \(m=1\). This is around 0.39, so that the original half width of the 1st order peak is reduced by almost 60% which is consistent with the numerical calculation results.

In summary we have covered the effects of reflectance of mirrors, phase shift on reflection, and the order of the device on transmittance of LC etalon device. More specifically these effects were analyzed with respect to the feasibility of LC etalons for transmissive field sequential displays. When phase shift on reflection was considered negligible, 1\textsuperscript{st} order etalon device with high birefringence LC (\(\Delta n\sim.5\)) was shown to be able to modulate \(\sim25\text{nm}\) spectrum over the visible spectrum ranging from 475nm to 640nm (see Figure 3.3). If LED backlight with spectrum width around 25nm is used, then transmittance of such device would be at most 40% because of the polarization loss.

In a real device with dielectric mirrors, phase shift on reflection causes limited tunability and phase dispersion causes reduction in half width of transmission peaks. If these drawbacks are mitigated in the device design and if it is made polarization
independent, then single cell transmissive design of LC etalon device makes it very viable candidate for pico-projection display devices compared to existing projection technologies. If successfully realized this device can have applications in mini pointer projectors that can be incorporated into almost any mobile devices such as cell phone, PDAs, and ultra portable computers.

3.2 Device Design

In this section we will outline the basic principles of etalon device what we consider for projection applications and analyze tradeoffs involving optimization of important variables such as transmittance, contrast, and switching speed of LC etalon. The basic equation for an etalon transmittance is shown in Eq. (3.1) where the optical effects of substrate, as well as the alignment and conducting layers are ignored\(^6\). Let us consider the case where etalon mirrors are identical and the phase change on reflection is negligible. In this case condition for transmission maximum is given by,

\[
\delta = 2\pi n_0 d / \lambda = m\pi \quad \text{where} \quad m = 1, 2, 3, \ldots .
\]

The width of transmission maximum is determined by the order of the peak and by the reflectance of the mirrors. Half width of transmission maximum (width at which transmission is half of its maximum value) is proportional to

\[
2(1 - R) / R \quad \text{where} \quad R \text{ is the mirror reflectance and it gets narrower as the order of the peak increases.}
\]

Free spectral range or the distance between adjacent transmission peaks can be derived from equation for transmission maximum to be \( \Delta \lambda \sim \lambda (m+1) \) which means that as the order of the etalon increases distance between successive peaks decreases.
If the phase change on reflection from the etalon mirrors is not negligible, then the equation for transmission maximum takes a new form, \((\phi_a + \phi_b)/2 - \delta = m\pi \ m = 1,2,3, \ldots\) where \(\phi_{a,b}\) is the phase change on reflection from the mirrors. In the case of dielectric mirrors phase change on reflection is wavelength dependent, because of the fact that optical thickness of dielectric layers is wavelength dependent. Such phase dispersion of the reflected light diminishes the tuning effect of changing the cavity index. Extensive discussion on details of effects of the phase change on reflection from dielectric mirrors on etalon transmittance is available in Macleod's thin film optics text^{61}.

In designing a LC etalon we optimized the order of the etalon, reflectivity of the mirrors, and birefringence of the LC material to get high efficiency transmission. It is advantageous to have lowest possible order device, because as the order increases width of transmission peak narrows. In a LC etalon device with field sequential illumination sources such as light emitting diode (LED), transmission peak must be wide enough to efficiently transmit as much of the emitted light as possible. Besides the order of the etalon, mirror reflectivity can be decreased to widen the etalon transmission peak, however low mirror reflectivity reduces the contrast of the device, because of the increased light leakage in the dark state.

Tunability is achieved in a LC etalon device by change in effective index of the mirror cavity with an application of external electric field. If the phase change on reflection from the mirrors is negligible, then the tuning range of transmission peak from wavelength \(\lambda_1\) to \(\lambda_2\) is related to the ratio of effective indices as \(n_1/n_2 = \lambda_1/\lambda_2\).
Consequently birefringence of the LC material is the limiting factor in etalon tunability. Most commercially available materials have birefringence less than 0.3\textsuperscript{62} although birefringence as high as 0.6 has been reported\textsuperscript{63}. The required ratio of LC indices for tuning the lowest order transmission peak over the visible spectrum ranging from 450 to 650 nm is \( \sim 1.4 \) which is too high for commercially available materials. However, the requirement for high birefringence is less severe in high order etalon with multiple transmission peaks in the visible spectrum. In such device transmission peak that is closest to the wavelength region of the illumination source is tuned in for the bright state and gray scale is generated by partially moving the same peak out of the wavelength region.

In a LC etalon if the director configuration through the cavity is co-planar, then only light polarized in the plane of the director experiences a change in the effective index as voltage is applied to the device. In this case, a polarized light source such as laser is used for the LC etalon device. In the case of un-polarized light sources such as LEDs, polarization sensitivity must be reduced for high efficiency transmittance. There have been three major developments in reducing polarization sensitivity of a LC etalon. These are polarization diversity technique\textsuperscript{64}, addition of birefringent retarders within the etalon cavity\textsuperscript{34,35}, and the use of 90\textdegree twisted nematic (TN) LC in the etalon cavity\textsuperscript{33}. The simple and cost effective structure of a 90\textdegree TN LC etalon makes it the most viable choice for our device. However, polarization insensitivity works only in the high field regime for 90\textdegree TN LC etalon, and as a result tunability is considerably reduced\textsuperscript{33}. In this chapter we
will use a new technique that uses an intermediate twist ($\pi/2 < \phi_{\text{twist}} < 2\pi$) LC etalon to achieve polarization independent LC etalon operation.

As previously noted, the switching speed of LC etalon must be fast enough to suppress the image breakup effects in field sequential devices. The low cell thickness required for an etalon device, as compared with a polarization rotation device, is expected to dramatically increase the switching speed because of the inverse relationship of switching speed and the square of the thickness of LC.

In summary, we have probed the main variables that need to be optimized in LC etalon device in order to achieve high brightness, sufficient tunability, and fast switching speed. Although a low order etalon has many advantages such as wide transmittance bandwidth and fast switching speed, birefringence of LC material limits the tunability in low order devices. However, we expect that increasing the order of the device will allow sufficient tunability in the visible spectrum. If the illumination light source has narrow spectral emissions like that of laser, then the high order etalon with narrow transmittance bandwidth fits well in compact high brightness projection system. However in a LC etalon device with unpolarized wide bandwidth light sources such as LEDs, the order of the etalon must be optimized to achieve the best transmittance efficiency.
3.3 Numerical Model

3.3.1 Methods

In this section we will implement our basic design concepts and show the path toward an optimized device through numerical modeling. We used software previously developed by Berreman to calculate the liquid crystal director configuration in the etalon cavity\textsuperscript{65}. We assume infinite anchoring energy such that LC molecules on substrate surfaces are fixed permanently with $2^\circ$ pretilt angle. The numerical method for dynamics of director configuration which assumes negligible angular momentum change, and inertia is based on the Erickson and Leslie\textsuperscript{66,67,68,69,70} hydrodynamic equations that take into account the “backflow” effects\textsuperscript{47}. Switching and relaxation speeds of LC were determined by calculating the time it takes for polar angle of the director in the middle of the cavity to reach 90\% of its final value.

Once the LC director configuration is known, we used Berreman’s 4x4 matrix method to calculate the optical transmittance of the LC etalon device\textsuperscript{45,46}. This method is a direct solve of Maxwell's equations in one dimension where individual optical elements are represented by 4x4-matrix that depends on the dielectric and other optical parameters of the material. The liquid crystal layer was discretized into 40 uniform layers. Transmittance of unpolarized light source was calculated by averaging the individual transmittances of orthogonal polarizations of light. We ignored the glass substrates, conducting and alignment layers in the optical calculations, because their effects can be minimized and or compensated in a real device.
For the particular LC material modeled (BL009 from Merck) the material parameters are: $n_e=1.81$, $n_o=1.53$, $\varepsilon_{//}=21$, $\varepsilon_{\perp}=5.5$, $\gamma\approx 0.370\text{PaS}$, $\eta_1\approx 0.452\text{PaS}$, $\eta_2=0.0755\text{PaS}$, $\eta_3=0.134\text{PaS}$, $K_{11}\approx 14.6\text{pN}$, $K_{22}\approx 7\text{pN}$, and $K_{33}\approx 29.9\text{pN}$. For the dielectric mirrors, we used a stack of alternating TiO$_2$ and MgFl layers. We modeled TiO$_2$ and MgFl as Cauchy materials of which the refractive indices are given by, 

$$n = A + B/\lambda^2 + C/\lambda^4,$$

where $A$, $B$, and $C$ are $2.17$, $3.43\times 10^4$, $4.02\times 10^9$ for TiO$_2$ and $1.35$, $4.07\times 10^3$, $0$ for MgFl. Dispersion effects of the LC material were ignored in the calculation and instead were discussed in later sections.

### 3.3.2 Results

We start by modeling a mirror with total of 6 dielectric layers of alternating high and low indices of which half are tuned to be quarter wave thick at 550 nm and the other half at 525 nm. Note that this is a very simple design and that the performance of the device can be improved with more refined mirrors. The modeled mirror reflectivity is shown in Figure 3.8.
Figure 3.8: Optical structure of the mirror is $n_H n_L n_H n_L n_H n_L$, where $n_H = n_L = 2.27, n_e = n_o = 1.38$ and $d_H = 60.6\text{nm}, d_L = 99.6\text{nm}, d_H' = 57.8\text{nm}, d_L' = 95.1\text{nm}$. Reflectance was normalized by the total unpolarized incident light intensity.

The LC etalon device modeled in this section consists of pair of these mirrors on either side of a tunable LC layer such that the optical structure of the device can written as $n_H n_L n_H n_L n_H n_L n_L n_H n_L n_H n_L n_L$. This device will be modeled using the methods and material parameters discussed earlier. In comparing the results in this section, the low and high index states will now be referred to as the “high field/unwound” and “low field/twist” states because an increased electric field will reduce the effective index of the LC to be closer to the ordinary index.

We would like to find the lowest order device that can modulate the desired wavelength regions, while using reasonable values for the LC birefringence. Etalon with 0.26 $\mu$m thick non-twist LC cavity has 1st and 2nd order peaks in the high field state and
a single 2nd order peak in the low field state (see Figure 3.9). The effect of increasing the cavity thickness to 0.35 \( \mu \text{m} \) is shown in Figure 3.10, where 2nd and 3rd order peaks appear in the low field state and a single 2nd order peak in the high field state. If the light source is polarized, then this is the lowest order device that could be considered to modulate wavelengths in the red, green, and blue area of the visible spectrum. We note here that if the light source is monochromatic, then high contrast can be achieved by reducing the dark state transmission with high reflectivity mirrors.

![Transmittance of etalon with 0.26 \( \mu \text{m} \) LC cavity for a light source polarized linearly and along the alignment axis of the director in the plane of the cell. The shaded region corresponds to the range of location of the 2nd order peak as a function of voltage. Transmittance was normalized by the total polarized input light intensity.](image)

Figure 3.9: Transmittance of etalon with 0.26 \( \mu \text{m} \) LC cavity for a light source polarized linearly and along the alignment axis of the director in the plane of the cell. The shaded region corresponds to the range of location of the 2nd order peak as a function of voltage. Transmittance was normalized by the total polarized input light intensity.
Figure 3.10: Transmittance of etalon with 0.35 µm LC cavity for a light source polarized linearly and along the alignment axis of the director in the plane of the cell. The shaded regions show the tuning range of the 2nd and 3rd order peaks as a function of voltage. Transmittance was normalized by the total polarized input light intensity.

For un-polarized light sources high efficiency can be achieved with polarization independent LC etalon transmission. The authors have published a novel method to reduce polarization sensitivity using a twisted director profile\textsuperscript{58}. Considering the methods explained in that paper, we find that a device like that modeled in Figure 3.10, but with a 165° twist, demonstrates polarization independent operation as shown in Figure 3.11. Note that in this figure the plots are normalized to incident un-polarized light. In this case, the effective index of refraction in the low voltage state is not as high as in the case of a uniform device, so the tuning range of the 2nd order peak is significantly reduced. As
the voltage applied to the device increases the 2nd order peak splits into two polarization peaks and comes back together at high fields at around 540 nm.

Figure 3.11: Transmittance of etalon with 0.35 µm 165° twisted LC cavity for unpolarized light source. The shaded region shows the tuning range of the 2nd order peak as a function of voltage. Note that peak splitting occurs in this area. Transmittance was normalized by the total un-polarized input light intensity.

Improvements can be made by going to next higher order device where 2nd and 3rd order peaks are present in the unwound state and a single 3rd order peak is present in the twist state (see Figure 3.12). However, tunability is still not sufficient to cover the visible region ranging from 450nm to 650nm, so the order of the device is increased such that 3rd, 4th, and 5th order peaks are present in the twist state and 3rd and 4th order peaks in the unwound state (see Figure 3.13).
Figure 3.12: Transmittance of etalon with 0.45µm 169° twisted LC cavity for unpolarized light source. The shaded region shows the tuning range of the 3rd order peak as a function of voltage. Note that peak splitting occurs in this area. Transmittance was normalized by the total un-polarized input light intensity.
Figure 3.13: Transmittance of etalon with 0.63μm 165° twisted LC cavity (n_o=1.53, n_e=1.81) for unpolarized light source. The shaded regions show the tuning range of the 3rd and 4th order peaks as a function of voltage. Note that peak splitting occurs in this area. Transmittance was normalized by the total un-polarized input light intensity.

At such high order transmittance peaks are too narrow for broadband light sources and 3rd and 5th order peaks have low transmittances as a result of polarization splitting. Instead of increasing the order of the device, tuning range can be increased with consideration for higher values of the birefringence than is available with standard, commercially available, LCs. It has been shown that LC mixtures with birefringence values as high as 0.6 can be considered\(^6^3\). As an example of the effect of a higher birefringence value, we increased the LC birefringence to be 0.4 (n_o=1.5, n_e=1.9) for the same device as in Figure 3.12 (see Figure 3.14).
Figure 3.14: Transmittance of etalon with 0.45 µm 159° twisted LC cavity (n_o=1.5, n_e=1.9) for unpolarized light source. The shaded region shows the tuning range of the 3rd order peak as a function of voltage. Note that peak splitting occurs in this area. Transmittance was normalized by the total un-polarized input light intensity.

LC dynamics modeling software described in the previous section, predicts switching time of the etalon in Figure 3.12 to be ~0.1 ms when 5 V is applied, and ~0.3 ms when 5 V is removed. In section 3.5 characteristics of the etalon shown in Figure 3.14 as a consideration for application in field sequential projection system will be discussed.

3.4 Discussion

Phase dispersion effect of the dielectric mirrors causes the argument of sine function in Eq. (3.1) to vary much faster as function of wavelength and lowers the tuning range of the etalon. Figure 3.9 shows that although LC index switches from 1.8 to 1.5, the 2nd order transmission peak moves from 510 nm to only 470 nm instead of 425 nm predicted when phase change on reflection is negligible (1.5x510/1.8). For the application considered here, this leads to the need to increase the order of the device.

LC etalon modeled in Figure 3.10 achieves requirements of tunability in the visible spectrum for polarized light sources such as laser. For example, if polarized RGB laser light sources centered at wavelengths 600, 540, and 460 nm are available, then this
device would be able to field sequentially modulate them. Since narrowing of the half width of transmission peaks will not cause light loss for monochromatic light sources, the reflectivity of the mirrors can be increased above 80%. In this case contrast of the device is significantly increased, because of the reduced transmission in the dark state. For example, if the dielectric mirror has one more layer of high index material such that the optical structure of the etalon becomes $n_H n_L n_H n_L n_H n_L n_H n_L n_H n_L n_H n_L n_H n_L n_H n_L n_H n_L n_H$, then calculated contrast for the peak wavelengths 600, 540, and 460 nm is around 150, 900, 100 respectively.

In the case of unpolarized light sources, polarization insensitive operation is required for maximum efficiency. This can be achieved with twist LC etalon where the polarization sensitivity is reduced for the transmission peak in the low voltage state. It was shown by the authors that if the relative phase difference between the eigenmodes in intermediate twist LC etalon is integer multiple of $2\pi$, then the resonance condition for polarization independent transmission peak is satisfied$^{58}$.

Figure 3.11 shows that the tunability is reduced for the 2nd order polarization insensitive device, because in the twist state mode mixing results in averaging of ordinary and extraordinary indices. We note here that for the 3rd order device, it is possible to modulate the RGB spectra if the birefringence of the LC is increased to 0.4 ($n_o=1.5$, $n_e=1.9$). Very high birefringence LCs is subject of numerous researches and recently LC with $\Delta n \approx 0.6$ and stable nematic phase at room temperature has been reported$^{63}$. Therefore, for demonstration purposes we modeled 3rd order etalon with LC that has the
same properties as BL009 except the birefringence is increased to 0.4. Figure 3.14 shows the transmittance spectrum of field on and off states where polarization insensitive peaks at wavelengths 480, 525, and 625 nm are present. Half width of transmission peaks is around 15 nm. In a real device dispersion effects will reduce the LC indices slightly for longer wavelength region. This causes the red transmission peak to move further towards shorter wavelengths. Although currently the contrast of the device is low, there are two straightforward fixes that can improve it significantly. First, increased tunability with higher birefringence material can shift the green peak closer to 550 nm and reduce the dark state transmission in the red region. Second, if better designed dielectric mirror with no reflectance drop in the blue region is used, then the contrast will be improved with reduced dark state transmission in the blue region.

We have demonstrated experimental etalon with high reflectivity mirrors where thickness is uniform within few nm across 1mm region. In real devices where the pixel size is on the order of several tens of μm the variation becomes even smaller. In addition, when reflectivity of the mirrors is reduced for wide bandwidth transmission, such peak movement is negligible in comparison to the transmission bandwidth. Similar effects can be observed for obliquely incident light where transmission peaks blue shift. However, this effect is almost negligible, because the total thickness of the device is less than 1.5 μm thick. In our calculation 10° off axis light caused the transmission peaks to shift only by ~2 nm.
Inherently thin, sub-micron design of LC etalon device has fast switching time compared to typical half-wave LC retarders and LC light modulators in commercial displays, because of the inversely proportional relationship between the square of the thickness and the LC relaxation time. In recent publications it was shown that this is true for LC cells with thicknesses as low as 1µm in the presence of strong anchoring condition\textsuperscript{71}. Although in an optimized LC etalon device where the thickness is around 0.5 µm we expect switching time to be slightly longer than the modeled results due to the finite anchoring condition, it is still much faster than the threshold time of 1msec. However, for thin cells effective birefringence of the LC material drops as a result of surface roughness and irregularities. Observed drop in the order parameter is within 30 nm to the surface and in cells with thickness around 500 nm this effect is small\textsuperscript{72,73}. Nevertheless even small decrease in the effective birefringence would lower the tunability and as a result increases the required LC birefringence.

\section*{3.5 Summary}

Previously, LC etalon based devices have not been considered for a compact multi-wavelength modulator. We have shown that a polarization sensitive 2nd order LC etalon can be used in projection devices with polarized light sources such as lasers. In this case near lossless transmittance and high contrast ratio can be achieved.

We also demonstrated a polarization independent 3rd order LC etalon that can modulate the red, green and blue wavelength regions. Polarization insensitivity was
achieved by having intermediate twist LC structure where phase difference between the eigenmodes was tuned to be $2\pi$.

Verbal communication with a product manager from OSRAM LED division resulted in a good prospect for LED light sources with narrow bandwidth light spectrum. According to Francis Nguyen (Sr. Product Marketing Manager, LED Products, Opto Semiconductors, OSRAM) current line of green LEDs have FWHM around 25nm and within a year it will decrease down to 15nm with 2X luminous brightness enhancement. Although current commercial LEDs have half width around 35nm (see Figure 3.15), we can expect their next generation LEDs to have narrower more spectral colors.

![Figure 3.15: Emission spectrum of OSRAM LEDs (Golden Dragon Argus)]
Target application of transmissive LC etalon device would be in portable pico-projection displays. Recently there has been lot of activities in pico-projection areas with the advent of laser projection displays and miniature DLP devices. Currently there are three well established projection display technologies available. These are 3LCD system, DLP system, and LCOS system. DLP and LCOS systems work in reflective mode, whereas 3LCD works in transmissive mode. Those that work in reflective mode require a complicated optical setup which increases the size of the device making pico-projection device less attractive. As for 3LCD, it works by splitting light beam into 3 primary components each of which is modulated by 3 separate TFT LCDs before combing them into single beam that is projected through optical array. Although use of the polarization conversion element increases the brightness 1.5 times that of polarization dependent devices, it still loses around 30% of the light through this element. In addition, beam splitters and light guiding elements required for multiple beam modulation increase the size and complexity of the device to the degree that is not acceptable for pico-projection devices. As far as the size and complexity of the device go, LC etalon has many advantages over these devices because of its single cell transmissive design.
3.6 Appendix 3A - Dielectric Mirror Phase Shift on Reflection and Phase Dispersion Effects

For an assembly of q layers of thin film stack, output field can be calculated as inner multiplications of 2x2 characteristic matrices of individual layers times the input field (see Eq. (3A.1)).

\[
\begin{pmatrix}
E_{out} \\
H_{out}
\end{pmatrix} = \left\{ \prod_{r=1}^{q} \begin{pmatrix}
\cos(\delta_r) & i \sin(\delta_r)/\eta_r \\
i \eta_r \sin(\delta_r) & \cos(\delta_r)
\end{pmatrix} \right\} \begin{pmatrix}
E_{in} \\
H_{in}
\end{pmatrix}
\]

Eq. (3A.1)

If we normalize Eq. (3A.1) by dividing both sides by E_{in}, then

\[
\begin{pmatrix}
E_{out} / E_{in} \\
H_{out} / H_{in}
\end{pmatrix} = \left\{ \prod_{r=1}^{q} \begin{pmatrix}
\cos(\delta_r) & i \sin(\delta_r)/\eta_r \\
i \eta_r \sin(\delta_r) & \cos(\delta_r)
\end{pmatrix} \right\} \begin{pmatrix}
1 \\
1 / \eta_{in}
\end{pmatrix}
\]

Eq. (3A.2)

Where, \( \eta = N \sqrt{\frac{\varepsilon_0}{\mu_0}} \). It's useful to define optical admittance, \( Y = \frac{C}{B} \), through which reflectance of an assembly of thin film layers can be calculated. Graph of admittance locus is used to visually interpret effects of additional layers of materials on reflectance of the assembly. For example, loci for dielectric layers take the form of a series of circular arcs, each corresponding to a single layer, which are connected at points corresponding to interfaces of two layers.
Figure 3A.1: Normalized admittance diagram of dielectric stack. First 1/4\(\lambda\) layer traces half circle from (1, 0) to (5.1, 0) and 2nd 1/4\(\lambda\) layer of the same material completes the circle back to (1, 0), whereas 2nd 1/4\(\lambda\) of low index material traces bigger semicircle to (0.37, 0).

Figure 3A.1 shows an admittance diagram of a stack of 1/4\(\lambda\) dielectric layers. Admittance loci starts from (1, 0) and traces semicircle in clockwise direction (in solid blue line) as thickness of the dielectric layer increases. Semicircle is completed at (5.1, 0) when dielectric layer reaches 1/4\(\lambda\) thickness. If the same dielectric material layer continues to increase to 1/2\(\lambda\) thickness, then loci will end where it started at (1, 0) signifying admittance matching for 1/2\(\lambda\) layer (see dashed red line). If low index 1/4\(\lambda\) material is put on top of 1st 1/4\(\lambda\) layer, then semi circle with larger radius will be traced to (0.37, 0). Structure of these two designs is shown in Figure 3A.2
Figure 3A.2: Optical structure of two stacks described in fig. 7. In the first stack two $\frac{1}{4}\lambda$ layer of same dielectric material makes $\frac{1}{2}\lambda$ layer which transmits the entire incident light at that wavelength. However, in the 2nd stack two layers of $\frac{1}{4}\lambda$ materials of high and low indices result in partial reflection of the incident light.

An important point to note in these diagrams is that admittance locus is drawn for single wavelength at which the thickness of the dielectric layers is calculated. For example, loci in Figure 3A.1 are true only for wavelengths at which the dielectric layers are $\frac{1}{4}\lambda$ thick. For other wavelengths terminal points of each locus would not terminate on the real axis since at that wavelengths dielectric layers are no longer $\frac{1}{4}\lambda$ thick. For instance, at wavelength which the 1st layer is $\frac{1}{4}\lambda$ thick locus ends at (5.1, 0), whereas for longer wavelengths locus would not be able to trace the entire semicircle, instead it will end somewhere above the real axis. Likewise for shorter wavelengths, locus will over trace the semicircle and will end somewhere below the real axis since at these wavelengths the
dielectric layer is more than $1/4\lambda$ thick. The phase on reflectance can also be approximated from the admittance diagram.

Figure 3A.3 shows the contours of constant phase shift (in solid black line) that makes the boundaries between four quadrants. Constant phase shifts 0, $\pi/2$, $\pi$, and $3\pi/2$ are these boundaries, and are represented by portion of the real axis and the circle centered on the origin and passing through admittance of the incident medium. If the exit admittance ends on a real axis to the right of incident admittance, then the phase shift would be exactly $\pi$. Likewise, if the exit admittance is on a real axis to the left of incident admittance, then the phase shift is 0. As before, these constant phase shifts are true only for single wavelength at which the dielectric layers are $1/4\lambda$ thick and deviates as the wavelength
moves farther away from the center wavelength. For example, at the center wavelength phase shift would be exactly $\pi$ for dielectric layer that is represented by blue line. However, for this same dielectric layer, phase shift would increase as the wavelength gets longer (goes into the 3rd quadrant) and would decrease as the wavelength gets shorter (goes into the 2nd quadrant).

Figure 3A.4 shows the reflectance curve of two dielectric mirrors differing only by 1 layer of $1/4\lambda$ material. Mirror ending with low index dielectric layer next to the incident medium (air in this case) has much lower reflectance than the mirror ending with high index material.

![Reflectance spectrum of dielectric mirrors](image)

Figure 3A.4: Reflectance spectrum of dielectric mirrors. Low reflectance mirror optical structure, $G(HL)^2A$, where $G$-glass substrate, $n_H=2.27$, $n_L=1.38$, dielectric layers are $1/4\lambda$ thick at 525nm for 2 layers next to glass substrate, and 550nm for other 2 layers. High reflectance mirror optical structure, $G(HL)^3HA$, where $G$-glass substrate, $n_H=2.27$, $n_L=1.38$. The reflectance spectrum shows the low reflectance mirror having a lower reflectance compared to the high reflectance mirror across the wavelength range from 450nm to 650nm.
nL=1.38, dielectric layers are 1/4λ thick at 525nm for 2 layers next to glass substrate, and 550nm for other 3 layers.

Reflectances of two mirrors in Figure 3A.4 can be explained easily by the admittance diagram of those mirrors which is shown in Figure 3A.5. In Figure 3A.5, admittance for both mirrors starts at (1, 0) on the real axis and traces out semi circles for each successive layer of 1/4λ dielectric material. Admittance of a mirror that has 4 layers of material ends at (0.1366, 0), whereas that of 5 layered mirror ends at (37.72, 0). Reflectance of these mirrors at 550nm wavelength can be calculated using Eq. (3A.3).

\[
R = \rho \rho^* = \left( \frac{n_{air} - Y}{n_{air} + Y} \right) \left( \frac{n_{air} - Y}{n_{air} + Y} \right)^* 
\]  

(3A.3)

For 4 layered mirror, \(R=\left[(1-0.1366)/(1+0.1366)\right]^2=0.58\), whereas for 5 layered mirror, \(R=\left[(1-37.72)/(1+37.72)\right]^2=0.90\). This result is in good agreement with the numerical calculation result shown in Figure 3A.4.
Figure 3A.5: Normalized admittance diagram of dielectric mirrors. 4 layer mirror starts from 1.0 (admittance of air) and ends at \( Y = 0.1366 \), whereas 5 layer mirrors starts from the same place and ends at \( Y = 37.72 \).

It is time now to investigate the effects of phase change on reflection in the performance of LC etalon devices. We have seen in the admittance diagrams that there will be a wavelength dependent phase change for light as it reflects from the dielectric mirror surfaces. It was shown previously that when the phase shift on reflection is neglected, transmittance maximums occur for \( \delta = m \pi \), where \( m = 1, 2, 3, \ldots \), and minimums halfway between these values. However, this no longer works when the phase shift, \( \phi \), is not negligible. More specifically, transmittance maximums now occur when the following condition is met,

\[
\frac{\phi_s + \phi_b}{2} - \delta = q \pi \quad \text{where} \quad q = 0, \pm 1, \pm 2, \ldots \quad (3A.4)
\]
It is clear from this expression that effect of phase changes is to shift the positions of transmittance peaks. If dielectric layers of a mirror stack are $1/4\lambda$ thick, then phase shift on reflection is 0, or $\pi$ depending on the admittance of the dielectric layers relative to the surrounding medium. For example, at the center wavelength a mirror with high index material next to air interface results in $\pi$ phase shift, whereas low index material results in 0 phase shift. However, in most cases phase shift for transmittance peaks is not exactly equal to but is close to 0 or $\pi$, because of the frequency dispersion.

When low order etalon devices with dielectric mirrors were simulated, phase change on reflection caused the peaks to shift to longer wavelength regions. In addition, phase change dispersion reduces the half-width of these peaks. We already know that the half width of peaks gets wider when transmittance of etalon gets bigger (decreased reflectivity mirrors). It’s also known that transmittance increases and decreases as function of wavelength because of the phase factor within the sine function (see Eq. (3.1)). When phase change on reflection from the mirrors enters the sine function, the rate of change of transmittance relative to the wavelength gets larger. This is a direct result of dispersion of phase changes on reflection. Faster variation of transmittance of the etalon relative to the wavelength causes the transmittance peaks to be sharper. This can be described by simple rise in the slope of transmittance relative to the wavelength ($dT/d\lambda$).

The effect of phase change dispersion on the half width of transmittance peaks can be calculated analytically. If two mirrors are identical and if there is no absorption in the mirrors, then following condition is met when transmittance is 0.5,
\[ F \sin^2 (\phi - \delta) = 1 \quad \text{where} \quad F = \frac{4R}{(1-R)^2}. \quad (3A.5) \]

Note that phase changes from two mirrors are identical and equal to \( \phi \). For small deviations from the peak wavelength (\( \lambda_0 \)),

\[ \delta = m \pi (1 + \Delta g) \quad \text{and} \quad \phi = \phi_0 + \frac{d\phi}{dg}\Delta g \quad (3A.6) \]

Where, \( g = \frac{\lambda_0}{\lambda} \). Note that at the peak wavelength, \( g = 1 \). If we substitute these results in to the equation of half peak, then

\[ F \sin^2 \left( \phi_0 + \frac{d\phi}{dg}\Delta g - m \pi (1 + \Delta g) \right) = F \sin^2 \left( \frac{d\phi}{dg}\Delta g - m \pi \Delta g \right) = 1 \quad (3A.7) \]

Here, \( \phi_0 \) is close to 0 or \( \pi \), so it was neglected along with \( m\pi \), because of the nature of sine function. In addition, if we are in the neighborhood of peak wavelength, then \( \Delta g \) is small and we can approximate \( \sin^2(x) \sim x^2 \). As a result we can get the expression for the half width, \( 2\Delta g \), of the transmission peak to be,

\[ 2\Delta g = \frac{2}{m \pi F^{1/2}} \left( 1 - \frac{1}{m \pi \frac{d\phi}{dg}} \right)^{-1} \quad (3A.8) \]

From this expression we can see that the half width decreases with the increasing rate of phase change. This rate of phase dispersion can be calculated for dielectric stack of high and low index materials to be \( \frac{d\phi}{dg} = -\pi n_L/(n_H-n_L) \) (H. A. Macleod, Thin-Film Optical Filters, 3rd ed., Institute of Physics Publishing (2001) pg 273). Finally, the equation for the half width of transmittance peaks is calculated to be,
\[ 2\Delta g = \frac{2}{m \pi F^{1/2}} \left( \frac{n_H - n_L}{n_H - n_L + n_L/m} \right) \]  

If phase change on reflection is neglected, then expression inside the parenthesis becomes 1 and half width equation is reduced to simply \(2/m\pi F^{1/2}\). This term within the parenthesis reduces the half width by substantial amount for low order devices. For example, for 1\textsuperscript{st} order device with mirror indices 2.2, and 1.3, half width is reduced to only 40\% of the original value. If the order of the device increases, then expression in the parenthesis approaches 1, and as a result half width gets closer to its original value.

It can be concluded that the phase change on reflection has two distinct effects on transmittance peaks. First, phase change on reflection shifts the transmittance peaks and as a result we saw higher order peaks even when the physical thickness of the etalon was low. Secondly, dispersion of the phase change causes the transmittance peak to drop off faster and as a result half width of the peak gets decreased. This effect is dominant in reducing the overall transmittance bandwidths.
Chapter 4

Transmissive LC Etalon Demo Device

4.1 Introduction

Recently there has been a surge in interest in ultra-portable projection devices called the pico-projectors\textsuperscript{74}. These miniaturized projection systems can be both embedded or stand alone device and offers high optical efficiency and low power consumption. Currently LCoS-based optical modulator\textsuperscript{75} and MEMS based digital micro-mirror devices\textsuperscript{76} are the main technologies existent in this field. Both of these technologies are based on reflective projection optics and therefore have disadvantages compared to simpler transmissive device in terms of form factor and cost. In addition LCoS based devices use polarization sensitive liquid crystal (LC) light modulator which requires polarization recovery optics for high light efficiency.

The LC etalon-based light modulator, which has been used as optical switches in mainly telecom applications\textsuperscript{77}, can be designed for broad bandwidth, high transmission devices such as displays. In such applications, unlike conventional LC displays, etalon tunability is achieved directly by phase retardation rather than polarization rotation effects and as a result it can be made polarization insensitive. In the previous chapters we have shown a new method for producing a polarization-independent LC etalon transmission using twisted LC configuration and explored its possible applications in
display devices. It was shown in Chapter 3 that a 3rd order etalon device that can modulate red, green and blue LED light sources would require LC birefringence $\Delta n \sim 0.4$, twist angle $\phi \sim 160^\circ$, and cell thickness $\sim 0.45 \mu m$ in order to achieve polarization independent transmission.

In this chapter an experimental demonstration of an LC etalon based light modulator that is polarization independent, transmissive and has significantly fast switching time is presented. Fast switching time due to its sub-micron LC cavity makes possible field sequential operation while the polarization-independent LC effect results in high efficiency transmission for portable projection application. The basic properties such as transmission efficiency, contrast and switching of the experimental device is analyzed in terms of existing LED light sources. Moreover the potential for very high efficiency device based on narrow emission spectrum LED device is discussed as possibility for future devices.

In Chapter 3, we have shown a detailed assessment of transmissive LC etalon device. Specifically we considered a design of polarization independent LC etalon device that can selectively tune RGB spectrum of light as applied in pico-projection devices. Figure 4.1 and Figure 4.2 show the dielectric mirror reflectivity and the transmittance of LC etalon design which consists of pair of these mirrors, and a LC material with birefringence of $\sim 0.4$. 
Figure 4.1: Dielectric mirror reflectance curve. Optical structure of the mirror is \( (n_H n_L)^3 \)
where \( n_H \sim 2.27, \, n_L \sim 1.38, \, d_H \sim 60.6 \text{nm}, \, d_L \sim 99.6 \text{nm} \) (light incident from high index side)

Figure 4.2: Polarization independent transmittance of 3rd order etalon device (See Chapter 3.3 for the details of calculation)
This chapter consists of detailed reports on the assembly and performance of demo devices based on this design. First, we will present the design and fabrication of the dielectric mirrors, and then will cover brief analysis of an empty LC etalon in terms of thickness uniformity and transmission efficiency. We built two different demos, one filled with LC with birefringence ~0.3 (BL009 from Merck) and the other with very high birefringence material (~Δn~0.4). The chapter will conclude with the detailed explanations of the performance shortcomings of the demos and will offer solutions that could fix these problems. Chapter appendices include experimental setups and techniques involved in assembling the demo devices. Note that all of the transmittance curves in this chapter are normalized by the total incident unpolarized collimated light such that 1 corresponds to transmission of unperturbed light from the source to the detector unless otherwise stated.

4.2 Dielectric Mirrors

We have designed and fabricated low reflectivity dielectric mirrors at Genvac AeroSpace Inc (Cleveland, Ohio)\textsuperscript{78}. Reflectance and transmittance curves as well as the stack design of the dielectric mirror are shown in Figure 4.3.
For the numerical calculations we modeled the TiO$_2$ and MgF$_2$ as Cauchy materials for which the refractive indices are given by $n=A+B/\lambda^2+C/\lambda^4$ where A, B, and C are 2.1735, 3.428E4, 4.017E9 for TiO$_2$, and 1.35, 4070, 0 for MgF$_2$. Indices of ITO and BK7 are taken as 1.9 and 1.52 respectively. Figure 4.4 shows the modeled mirror reflectance compared with the experimental result. Although discrepancies exist between experimental and modeled results, they are outside the interested spectrum region (450nm to 650nm).
4.3 Thin Cell Fabrication

After putting SiOx post spacers and the alignment layer (see Appendix 4A for the details) on these mirrors we assembled a 0.45um etalon cell. Figure 4.5 shows the transmission spectrum of the cell measured over a 2mmx2mm region as a uniformity test. Etalon transmission peak shifts about 7nm over the 2mm region which means that the thickness is uniform within 10nm over this region.
Figure 4.5: Empty etalon transmission at several points suggests thickness uniformity within 10nm over 2mmX2mm region (1 corresponds to total incident unpolarized light)

Figure 4.6 shows the transmission of the modeled etalon compared with that of the experiment. In the model we included 30nm thick SiOx (n=1.45) alignment layers on top of the mirrors (see the modeled stack). In addition, 8% loss due to the reflection from air-glass interface is included in the model transmission. Note that the experimental transmittance is not nearly high as the modeled transmittance especially in the blue region of the spectrum.
Figure 4.6: Experimental and modeled transmittances of the empty etalon cell (d=0.45um, n=1.0 and I corresponds to total incident unpolarized light)

Figure 4.7 shows the transmittance of the modeled etalons with thicknesses 0.45um and 0.46um. The shift in the transmission peak is about 7nm as in the case of experimental etalon transmission which means that the thickness indeed is uniform within ~10nm (See Figure 4.5).
We filled the 0.45µm thick etalon cell with high birefringence chiral LC mixture which has Δn~0.29 (n_0~1.53, n_e~1.82). LC mixture contained ~92wt% BL009 and 8wt% left-handed chiral material S811 from Merck. Gradient cell was assembled in order to check the pitch of the material. Before filling the cell blue marks were used as fringe spacing which is approximately 0.25µm (see Figure 4.8). Marks were put next to the glue lines (dark dots below the marks) so that thickness would not change significantly upon filling the cell. Upon filling the cell texture was very non-uniform with zigzags in-between two boundary states. However, after N-I-N transition boundaries were more or
less clear and pitch of the material was determined to be ~1\(\mu\)m. Note that as the thickness increases texture gets darker in-between crossed polarizers.

Figure 4.8: Boundaries between different twist regions are clearly visible in the gradient LC cell

Cell 0101-4 has 50nm 30deg SiOx alignment on top of 50nm 90deg SiOx layer. No spacer was used and instead LC material was placed on one substrate and let the capillary forces hold the other substrate on it. Thickness was measured and fitted in the thinnest region just outside the boundary b/w LC and air gap and found to be ~0.43\(\mu\)m (See Figure 4.9).
In this region LC texture switched between bright and dark colors as the cell is rotated b/w crossed polarizers (See Figure 4.10). LC texture transitioned to another state at around 0.25µm away from the thinnest region (near 0.68µm). If the pitch of the LC material is 1µm, then LC should switch between 160° state and 340° state at around 0.69µm which is consistent with the experimental result.
However, there are couple reasons to think that the LC in the thinnest region is in 20° state and not in 160° state. First of all, this region almost acts as homogeneous LC in-between crossed polarizers. Second, video images taken during the filling process shows that leading edge of the LC material is always in this state even in thicker region (right image in Figure 4.11). In the thinner region the bright state stays stable whereas in the thicker region bright state is not stable and switches to dark region as the LC-air interface moves away from it (see arrow in right image in Figure 4.11). This can be explained as the LC fills it briefly takes the 20° state which is the lowest twist state and then it switches to higher twist states if it is unstable.
In order to check this, we assembled another cell (0104-1) with no spacers. This time one of the substrates was heated on hot plate to above 110°C and drop of LC was placed on it before the other substrate was put on it. Very distinct two states were formed upon cooling where one state surrounded the other state which was in the middle (See Figure 4.12).

Figure 4.11: Texture analysis in cell 0101-4

Figure 4.12: Two states in cell 0104-1
Cell thickness was measured just outside the LC-air interface and found to be near 0.44µm (See Figure 4.13). There was about 1 fringe between outer and inner region which means the inner state thickness was less than 0.2um.

Figure 4.13: Just outside the LC filled area in cell 0104-1

Texture of these two states in-between crossed polarizers are shown in Figure 4.14. The inner region switched between dark and bright states as the cells is rotated in-between crossed polarizers (See Figure 4.15).
Based on this analysis we're convinced that the brighter state in the thinner region is a $20^\circ$ twist state and darker adjacent state is a $160^\circ$ twist state. As for Figure 4.10 it seems that
brighter state just next to the LC-air interface is 1 fringe thinner than 0.43µm which means that the adjacent darker state is 1 fringe up (back to 0.43µm).

In order to get single twist state in the cell we have to fill the LC etalon in the isotropic phase. This will help keep the flow alignment from resulting in more than one twist states even when the cell thickness is relatively uniform. The next LC etalon was filled in isotropic phase (at 110°C) and was cooled to nematic phase once the cell was full and entrance sealed with epoxy. Figure 4.16 shows the experimental transmission curve of the LC etalon.

![Transmittance Vs Wavelength](image)

Figure 4.16: Transmittance of etalon filled with LC (Merck-BL009) material with Δn~0.3 (1 corresponds to total incident unpolarized light)
However, modeled transmittance shows good agreement with the experiment only if the LC thickness is 0.4µm not 0.45µm (See Figure 4.17). This means that the substrates got squeezed together during the filling process. This could be due to the fact that etalon was sealed before LC completely filled the empty region under vacuum and the atmospheric pressure kept the etalon plates tightly together. In addition, LC was filled into the etalon in isotropic phase and was cooled to nematic phase which could have contracted the distance between the plates.

Figure 4.17: Modeled (dashed) and experimental (solid) transmittances of etalon filled with \( \Delta n \approx 0.3 \) LC material \( (d_{\text{LC}}=0.4\mu\text{m}, n=1.53/1.81, d_{\text{SiO}_x}=30\text{nm}, n_{\text{SiO}_x}=1.45) \)

Note that the modeled etalon transmission peaks have much higher amplitude than the experimental transmission peaks. In the model we did not consider the effects of
dielectric mirror absorption and slight thickness variation in the cell. Both of these will cause the transmission peaks to reduce. Presence of absorption is evident in the empty etalon transmittance where the transmission peaks are much shorter than the modeled transmission peaks (see Figure 4.6). In addition, polarization peak splitting will also reduce the transmission peaks further in the LC filled etalon. More detailed explanations on these subjects will be covered in sections 4.5 and 4.6. Figure 4.18 shows the transmission spectrum of the LC etalon superimposed on the emission spectrums of RGB LEDs. It is our goal to increase the tunability of the etalon by using higher birefringence LC material so that the transmission peaks better match the LED emission spectrums.

Figure 4.18: Transmittance of 0.40µm cavity LC etalon (Δn~0.3) and emission spectrums of RGB LEDs (solid red-twist state at 0Volts; solid blue-unwound state at 9Volts)
4.4.2 LC Etalon with High $\Delta n$ Mixture

We obtained high birefringence LC mixture from Prof. L. C. Chien (Liquid Crystal Institute, Kent State University) of which the indices were measured previously using lasers with wavelengths 460, 488, 514, 633, 1064, and 1550nm$^{79}$. The mixture has high dispersion in the visible wavelength region (See Figure 4.19).

![Diagram of LC Indices Vs Wavelength]

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>$n_o$</th>
<th>$n_e$</th>
</tr>
</thead>
<tbody>
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<td>1.575</td>
<td>1.989</td>
</tr>
<tr>
<td>488</td>
<td>1.568</td>
<td>1.97</td>
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<td>514</td>
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<td>633</td>
<td>1.518</td>
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<tr>
<td>1064</td>
<td>1.518</td>
<td>1.818</td>
</tr>
<tr>
<td>1550</td>
<td>1.531</td>
<td>1.817</td>
</tr>
</tbody>
</table>

Figure 4.19: High $\Delta n$ (~0.4) LC mixture indices have strong dispersion in the visible region of the spectrum.

For the etalon model indices of the LC mixture as function of wavelength were calculated using Cauchy formula, $n=A+B/\lambda^2+C/\lambda^4$, where constants $A$, $B$, and $C$ were found by fitting the experimental indices$^{80}$. Figure 4.20 shows the fitted curves for ordinary (blue) and extraordinary (red) indices of the mixture for wavelengths ranging from 450nm to
650nm. In the low field twist state effective index of the LC etalon cavity is the average of the two indices. This average of the ordinary and extraordinary indices is shown in brown in Figure 4.20.

Figure 4.20: Dispersion of the fitted LC indices of the high ∆n LC mixture \((A_o=1.46, B_o=2.5e4, C_o=0; A_e=1.76, B_e=3.5e4, C_e=3e9)\)

Figure 4.21 shows the experimental and fitted birefringence of the LC material as function of wavelength.
Roughly 45mg of the high Δn mixture was mixed with ~4mg of chiral material S811 so that the composition of the mixture is ~92wt% and ~8wt% respectively. The mixture was placed in small vial and was placed on hot plate at ~100°C for few hours. Since mixing magnet was too large for the amount of the material, periodic shaking was applied so that the chiral would mix well with the LC.

The LC etalon cell was assembled using previously prepared substrates with 0.45μm thick SiOx posts. Figure 4.22 shows the experimental transmittance of an empty etalon compared to modeled transmittance in which cavity thickness was ~0.45μm. From here on we include absorption in the modeled transmittances. We do this by fitting the empty cell transmittance by varying the imaginary component of the SiOx refractive index and keeping it the same for LC filled etalon models.
Figure 4.22: Comparisons between modeled and experimental transmittances of an empty etalon (d=0.45µm, n=1.0, d_{SiOx}=30nm, n_{SiOx}=1.45-i0.03 and 1 corresponds to total incident unpolarized light)

In order to test the new material 0.45µm cell was filled in the vacuum chamber. During the filling procedure a small amount of material on a tip of metal spatula was applied to the etalon opening and the vacuum chamber was vented. After that the etalon was placed on hot plate at ~115°C so that LC mixture would fill in isotropic phase the rest of the way. When the LC etalon is about ~3/4 filled 5 minute epoxy was applied to the cell inlet and the etalon was removed from the hot plate. Optical textures of the LC mixture taken by digital camera after the filling process are shown in the next figure (Figure 4.23). LC texture was mostly uniform except for the dark voids near post spacers. Rotation under
crossed polarizers resulted in dark state in which bright lines resulting from the filling process is revealed (bright lines in the right image).

Figure 4.23: Optical textures of the LC in-between crossed polarizers

In fact continuous rotation between crossed polarizers yielded 5 darker states which is consistent with the previous findings of 160° twist LC texture (see Figure 4.15).
Figure 4.24: Optical textures of the LC in-between crossed polarizers

Figure 4.25 shows the transmission spectrum of the LC etalon in the field on and off states. It can be seen that the LC switching saturates at around 4V. In the 0V twist state there is a slight peak splitting at ~500nm region.
Figure 4.25: Experimental transmittance of high $\Delta n$ mixture filled etalon (1 corresponds to total incident unpolarized light)

Figure 4.26 and Figure 4.27 compares the experimental transmittances of the etalon to that of the modeled results for field on and off states respectively. However, as with the previous case the etalon cavity thickness contracted after the filling process. Fitting process yielded $\sim0.36\mu m$ thickness for the LC layer which is $0.09\mu m$ less than the empty etalon thickness.
Figure 4.26: Comparisons between the modeled and the experimental transmittances of the unwound state (d=0.36\mu m, d_{SiOx}=30nm, n_{SiOx}=1.45-i0.03)

Figure 4.27: Comparisons between the modeled and the experimental transmittances of the twist state (d=0.36\mu m, d_{SiOx}=30nm, n_{SiOx}=1.45-i0.03)
Figure 4.28 shows the transmission spectrum in the 450nm to 650nm window where there is no green transmission peak. The LC layer thickness must be increased further so that field-on and field-off transmission peaks align well with the RGB LED emission spectrums.

Figure 4.28: Experimental transmission of the LC etalon with high $\Delta n$ mixture shows an absence of green transmission peak in the visible region.

To do this we heated the LC etalon cell to $110^\circ$C on a hot plate and cooled it down to room temperature (over several hours) so that the cell would expand a little bit and keep the increased cavity thickness. This is precisely what happened as Figure 4.29 shows that the transmission peaks have shifted after the N-I-N transition.
Figure 4.29: Experimental transmission of LC etalon with high $\Delta n$ mixture after N-I-N transition (1 corresponds to total incident unpolarized light)

Indeed modeled transmission shows that the LC etalon cavity thickness is increased to ~0.4$\mu$m (See Figure 4.30). Finally, Figure 4.31 shows the experimental transmission of high $\Delta n$ LC mixture superimposed on the RGB LED spectrums.
Figure 4.30: Modeled (dashed) and experimental (solid) transmittances of the LC etalon for 0 field twist (blue) and 4V unwound (red) states ($d_{LC}=0.40\mu m$, $d_{SiOx}=30nm$, $n_{SiOx}=1.45-i0.03$)

Figure 4.31: Experimental transmittances (solid lines) of the LC etalon superimposed on emission spectrums (dashed lines) of RGB LEDs (1 corresponds to total incident unpolarized light)
4.5 Effects of Absorption

Although inclusion of reflection from the air-glass interface in the numerical model decreased the transmission peak to be closer to the experimental transmission, there is still noticeable discrepancy between the two. Experimental transmission peak at ~420nm is almost 30% lower than the modeled transmission whereas that at ~515nm is almost 10% lower than the modeled transmission (See Figure 4.6). This means that there is some absorption that is decreasing the light throughput of the etalon. Figure 4.3 shows that the absorption is almost negligible for the bare mirror substrates. As a result we measured the reflectance and transmittance of the mirrors with alignment layer and with post spacers. Figure 4.32 shows the reflectance/transmittance graph of a mirror with ~45nm SiOx alignment layer. First, as a result of the alignment layer the reflectivity of the mirror is decreased significantly (compare with Figure 4.3). But more importantly it can be seen that there is a slight absorption over the most of the spectrum, especially in the shorter wavelength region. The same measurement for a mirror with ~45nm SiOx alignment layer and 300µm pitch hexagonal close packed 0.45µm SiOx post spacers resulted in even more absorption (See Figure 4.33).
Figure 4.32: Transmittance/reflectance spectrums of Genvac mirror w/ 45nm SiOx alignment layer

Figure 4.33: Transmittance/reflectance spectrums of Genvac mirror with 45nm SiOx alignment layer and 300μm pitch 0.45μm height hexagonal close packed SiOx post spacers
As a result of SiOx absorption numerical model did not accurately predict the transmittance amplitudes. In order to take account of the absorption we increased the imaginary component of the SiOx alignment layer in the model until model agrees with the experiment. For 45nm thick SiOx alignment layer absorption coefficient of \( \sim 0.03 \) resulted in best fit (see Figure 4.34).

![Reflectance Vs Wavelength](image)

**Figure 4.34**: Modeled reflectance compared with the experimental reflectance with 45nm SiOx alignment layer and 300\( \mu \)m pitch 0.45\( \mu \)m height hexagonal close packed SiOx post spacers

However, as Figure 4.35 shows the modeled transmittance of the same substrate still has some discrepancies in comparison to the experimental data. Nevertheless within the
interested spectrum region ranging from 450nm to 650nm transmittance agrees quite well with the experiment.

Figure 4.35: Modeled T/R spectrum Vs Experimental T/R spectrum

With the absorption included in the numerical model we calculated the transmittance of the empty etalon cell and compared it with the experimental result (See Figure 4.36). Indeed, the modeled transmission with absorption looks much more like the experimental transmission.
4.6 Transmission Amplitude Reductions

In this section we would like to discuss the main causes of transmission amplitude reductions. Particularly we will investigate the different sources of losses and will analyze their effects in the transmission of LC etalon with high $\Delta n$ mixture. The main sources of the transmission losses are listed below:

1) Losses due to absorption of SiOx and scattering from the post spacers

2) Losses due to thickness non-uniformity

3) Losses due to voids or non-filled regions

4) Losses due to polarization sensitivity
- Twist state polarization sensitivity

- Unwound state polarization sensitivity

1) Losses due to absorption of SiOx and scattering from the post spacers

In section 4.5 we covered the losses due to absorption of the dielectric layers. Specifically, we experimentally measured the losses due to the SiOx alignment layers and the post spacers. Combined absorption losses of the mirrors from these two sources were about 3% in the spectrum region ranging from 450nm to 650nm (see Figure 4.35). However, in an etalon cell which consists of two of these absorptive mirrors transmission peak amplitudes are reduced much more than 3%. Scattering and absorption losses due to the SiOx alignment layers and the post spacers are readily seen in the transmission of empty etalon. Figure 4.36 shows that the transmission is about 70% at 420nm region and ~85% at 520nm region. Reflections from the air-glass interface accounts for about 8% of the losses which means that the absorption and scattering losses amount to ~20% at 420nm and ~5% at 520nm region. The only other source of transmission loss in an empty etalon is thickness non-uniformity.

2) Losses due to thickness non-uniformity

We briefly covered thickness non-uniformity of the etalon device in section 4.3. It was shown that over 2mmX2mm region thickness of the etalon varied by as much as 7nm. As
a result transmission peak of the empty etalon shifted by about 7nm (see Figure 4.5). In our measurement setup spectrometer measures transmission over the range of <1mm region and the transmission spectrum is averaged over this region. As a result of this averaging effect amplitude of the transmission peak shortens and the transmission half-width increases. However, this effect is not as severe in wide transmission peak etalons as it would be in etalons with very narrow half-width. Figure 4.37 shows the averaging effect of thickness non-uniformity where ~5nm gradient resulted in slight decrease in the transmittance amplitude. This shows that in our measurements transmission amplitude decrease due to thickness non-uniformity is almost negligible. In addition, decrease in amplitude is compensated by the increase in the transmission bandwidth and is not a loss in terms of light throughput as long as the LED emission spectrum is wide enough.

Figure 4.37: Average transmittance over 5nm gradient region results in slightly decreased transmission peak amplitude
3) Losses due to voids or non-filled regions

Even when the LC is filled in the vacuum there is a tendency to form small voids or unfilled regions as the cell is filled. Figure 4.38 clearly shows the voids (dark spots) alongside the uniform LC twist regions. From this figure we can estimate the percentage of void in the LC filled region to be around 15% which is quite high.

Figure 4.38: Appearance of voids (in dark) is clearly seen in the polarizing optical microscopy picture of the LC etalon.
Figure 4.39 shows the modeled transmission of LC etalon with 15% void and 85% LC region in the high field state (dark curve). It can be seen that as a result of void region transmission has rise in the 670nm region.

Indeed, experimental transmission has the same bump in 670nm region as shown in Figure 4.27. With the consideration of the void regions numerical model prediction of LC etalon transmission in the high field state is much closer to the experimental result as shown in Figure 4.40.
Figure 4.40: Comparisons between experimental and modeled LC etalon transmittances in the high field state.

Figure 4.41 and Figure 4.42 show the similar curves of modeled transmission that accounts for the void region in the twist state (0 field state) and comparisons with the experimental transmission.
Figure 4.41: Modeled LC etalon transmittance of 85% LC and 15% void region (d=0.36µm)

Figure 4.42: Comparisons between experimental and modeled LC etalon transmittances in the twist state
Figure 4.43 and Figure 4.44 show the modeled transmission that includes the void for thicker LC etalon (0.4µm) and its comparison with the experimental transmission. From looking at the increased transmission in 700nm region in the model which is inconsistent with the experimental result we can conclude that void in this case was not as much as 15%. Perhaps during the N-I-N transition voids coalesced and the transmission was measured in much more uniform LC region.

Figure 4.43: Modeled LC etalon transmittance of 85% LC and 15% void region (d=0.4µm)
Figure 4.44: Comparisons between experimental and modeled LC etalon transmittances in the high field state

4) Losses due to polarization sensitivity

Finally, we would like to discuss about the decrease in transmission amplitude as a result of polarization splitting. This happens in the twist state when the transmission peaks of different light polarizations do not coincide. Although in the high field state director aligns homeotropically there is still small polarization splitting, because director at the surface do not align with the field as a result of anchoring energy. In the experimental results polarization splitting causes the measured transmission peak to be wider and has shorter amplitude. For example, in Figure 4.27 experimental transmission peaks are shorter and wider than the modeled transmission peaks in LC etalon with 0.36µm thickness. Similarly in Figure 4.30 (d=0.4µm) experimental transmission peaks are quite a bit shorter and wider than the modeled results in the blue and green region.
4.7 Discussion

In Figure 4.33 transmittance and reflectance measurements of the dielectric mirror substrate reveal losses due to absorption. Combined absorption losses of the dielectric layers were about 3% in the spectrum region ranging from 450nm to 650nm. However, in the LC etalon which consists of pair of these mirrors absorption results in much higher percentage decrease in the transmission peaks. It is evident in reduced transmission peaks of the empty etalon as shown in Figure 4.36. It shows that the transmission is about ~70% at 420nm region and ~85% at 520nm region. Reflections from the air-glass interface accounts for about ~10% of the losses which means that the absorption and scattering losses amount to ~20% at 420nm and ~5% at 520nm region. In the numerical model we take account of the absorption losses by non-zero absorption coefficient in the SiOx alignment layer indices. Although more accurate model would need to consider the absorption coefficients for all dielectric layers and account for their dispersion effect, our simplified model is adequate in illustrating the effects of absorption.

Figure 4.45 shows the transmittance of the LC etalon in the high field unwound state and in the 0 field twist state. In this figure the transmittance plots are normalized by the total incident unpolarized light intensity and we would like to break down the causes for transmission losses. First of all ~8% of the losses is due to the reflections from the air-glass interface. As described earlier there are about 10% and 5% absorption losses for the
transmission peaks at 475nm and 520nm region respectively. That means there are about 10% reductions in the transmission peaks that are due to other reasons (see Table 4.1).

![Graph showing transmittance vs wavelength for an LC etalon in high field unwound and zero field twist states.](image)

Figure 4.45: LC etalon transmittances in the high field unwound and zero field twist states (1 corresponds to total incident unpolarized light)

<table>
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<th>Wavelength (nm)</th>
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<th>625nm</th>
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<tr>
<td>Remaining</td>
<td>9%</td>
<td>10%</td>
<td>10%</td>
</tr>
</tbody>
</table>

Table 4.1: Break down of transmission losses for the LC etalon
The remaining 10% losses are due to number of other variables such as peak splitting due to polarization sensitivity, thickness non-uniformity, and the LC material absorption etc. For example, when the individual transmittances of the orthogonally polarized eigenmodes traversing through the LC etalon do not coincide, the total transmittance becomes average of these transmittances which has wider spectral bandwidth but shorter amplitude. Polarization peak splitting in the high field state can result from residual birefringence at the surface where the director is not quite homeotropic, whereas in the zero field twist state breaking of the requirements for polarization independent transmission due to thickness and twist angle errors can have the same effect. Nevertheless as explained earlier such polarization peak splitting increases the transmission peak bandwidth at the same time reducing the amplitude and the overall light throughput is the same.

As a result of dielectric mirror stack the voltage across the LC layer is slightly less than the applied voltage. In order to calculate the voltage drop across the dielectric stack we assembled simple non-twist ECB cell filled with 5CB and measured the threshold voltage which then was compared to cell assembled with substrates without dielectric stack. From here the $V_{app}/V_{LC}$ ratio is determined to be around ~1.1. In addition, we approximated this ratio using Eq. (4.1)\textsuperscript{81} where $\varepsilon_{ins}$, $d_{ins}$ are the dielectric permittivity and thickness of the dielectric stack respectively.

$$V_{LC} = \frac{V_{app}}{1 + \frac{2\varepsilon_{LC}d_{ins}}{\varepsilon_{ins}d_{LC}}} \quad \text{Eq. (4.1)}$$
In the literature dielectric permittivities for TiO$_2$ in random orientation is 60-80, and for MgF$_2$ is \( \sim 8 \). We can calculate the effective dielectric permittivity of the stack by assuming each layer is capacitor in series with the others and knowing the total thickness of the stack. In this way we calculated \( V_{\text{app}}/V_{\text{LC}} \) ratio to be around \( \sim 1.8 \).

It was explained in chapter 3 that the modulator for field sequential color device needs to have a switching time less than 5msec and preferably \( \sim 1 \)msec in order avoid visual image breakups. Experimental results of the LC etalon show that the turn on time is around 120\( \mu \)sec whereas the turn off time is \( \sim 1.5 \)msec. Although turn on time agrees well with the numerical calculation results, turn off time is quite slow. This could be due to the fact that lack of pretilt in the alignment layer caused delayed response as a result of LC molecules twisting the wrong direction initially when applied field is removed. Numerical model would not be able to predict such delays and in addition assumptions for negligible angular momentum and fluid inertia could pose serious limitations for the model in extreme conditions due to sub-micron cell cavity and high electric field strength. For example, in sub-micron cell surface viscosity would have increasingly important effect on the switching dynamics whereas the model only considers LC bulk viscosity. Nevertheless switching speed is fast enough for most applications and we expect it to increase with subsequent devices.

Finally, we would like to comment on the transmission efficiency and the contrast of the device for RGB LED backlights. Figure 4.31 shows the current device transmission superimposed on commercial RGB LEDs. Currently etalon transmission
peaks are not so high as a result of dielectric stack absorption. In addition, it can be seen that the transmission peak of LC etalon is quite narrow in relation to the LED spectrum especially in the case of green LED. The half width of red, green, and blue transmission peaks of the etalon are around ~25nm, ~12nm, ~10nm respectively. On the contrary, widths for commercial R, G, and B LEDs are around ~25nm, ~30nm, ~25nm respectively\textsuperscript{83}. One way to increase the etalon transmission bandwidth is to have decreased mirror reflectivity. However, this causes the dark state light leakage increase which reduces the contrast of the device. It is also conceivable to reduce the order of the etalon device in order to increase the transmission bandwidths. Although LC material with birefringence as high as 0.6 is reported\textsuperscript{63} it is not sufficient to achieve spectral R, G, B transmission tunability for lower order etalon device\textsuperscript{84}.

The optimal device can be realized if the LED emission bandwidths are decreased so that it is similar to that of the etalon transmission bandwidths. For illustration purposes we have modeled etalon device with LC material birefringence ~0.6 and lossless dielectric mirrors. Figure 4.46 shows the numerical calculation results of the etalon device (solid curve) with higher birefringence material where the LC Cauchy constants are A_{o/e}=1.46/1.96, B_{o/e}=2.50E4/3.50E4, C_{o/e}=0/3.00E9. Note that the reflection from the air-glass interface is ignored, because in real devices it can be reduced with AR coatings. In order to calculate transmission efficiency of the LC etalon we modeled RGB LEDs with peak emission that coincides with the etalon transmission peaks. Approximate transmission efficiency for given FWHM LEDs were determined by the ratio of integrated areas of the etalon transmission and LED emission. For 15nm FWHM LEDs
(dashed curves in Figure 4.46) RGB transmission efficiency is nearly ~80%, ~65%, ~50%, whereas for 25nm FWHM LEDs it drops down to ~70%, ~55% and 40% respectively.

Figure 4.46: Modeled transmittances of the LC etalon for twist (solid blue) and unwound (solid red) states ($d_{LC}=0.38\mu m$, $\phi=145^\circ$, $d_{SiOx}=30nm$) and 15nm FWHM LED emissions (dashed curves)

In f/2 optical system collection angle of the incident light is ~14$^\circ$ and so it is important to have a wide viewing angle for the device in order to efficiently collect all the light emitted by the LED sources. We calculated the transmission spectrums for the twist and unwound states for 10$^\circ$ off-axis light (dashed curves in Figure 4.47) and the results indicate that the transmission peaks shift by about <5nm. There are two sources for the
viewing angle dependency which are the dielectric mirror component and the LC component. It is possible to reduce the viewing angle dependency by having more advanced dielectric mirror which has variable index profile as function of incident light angle and by inserting viewing angle compensation films on either side of the LC layer.

Figure 4.47: Modeled transmissions of LC etalon for twist (blue) and unwound (red) states ($d_{LC}=0.38\mu m$, $\phi=145^\circ$, $d_{SiO_x}=30nm$, incident angle = $10^\circ$ for dashed curves)

4.8 Summary

Experimental demonstration of a new type of polarization independent fast switching LC etalon modulator for LED backlit pico-projection application is presented. The etalon modulator utilizes for the first time a polarization independent LC configuration for select wavelengths previously proposed in the chapter 2. Fast switching
time and low driving voltage of sub-micron design of the LC etalon makes it ideal for the field-sequential portable projection application. Although the transmission efficiency is low if current commercial LEDs are used, it will be significantly improved as the LED spectral bandwidth narrows.

4.9 Appendix 4A - LC Cell Fabrication and Filling

Previously we developed a thin cell spacer technology where we coated 1.5-2µm thick photoresist on one of the cell substrates and patterned a hexagonal close packed grid of holes on them. Then the substrate was coated (evaporative) with certain thickness SiOx film before dissolving the photoresist in acetone using ultrasonic bath. Figure 4A.1 shows the optical microscopy image of the post spacers on a sample substrate.

Figure 4A.1: Hexagonally packed post spacers of SiOx
We were able to monitor the thickness of the SiOx posts using quartz detector filament in the coating chamber. The actual thickness was then calibrated by measuring the SiOx film thickness of a witness silicon wafer using J. A. Woollam Co., Inc spectroscopic ellipsometer.

Once post spacers were placed on one of the substrates, SiOx alignment layer was coated on both substrates. Alignment direction was along the long axis for one of the substrate and for the other substrate it made $20^\circ$ angle with the long axis (See Figure 4A.2). This configuration yields $\sim160^\circ$ twist angle for left handed chiral material (S811) doped LC mixture.

![Figure 4A.2: Alignment direction of the cell substrates](image)

Figure 4A.3 shows the actual placement of substrates on the deposition plate (left) and the diagram of deposition plate orientation with respect to the SiOx source (right).
Table 4A.1 shows the LC twist angle configuration as function of d/p values for the alignment layers described earlier. For example, in our case where the pitch of the LC mixture (8wt% S811 and 92wt% LC) is ~1 and the chiral is left handed, the twist angle would be 160° if the cell thickness is thicker than 0.19µm but thinner than 0.69µm.

Table 4A.1: LC twist angle for d/p values (left and right handed chiral materials)

<table>
<thead>
<tr>
<th>Handedness</th>
<th>d/p</th>
<th>Twist Angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LH</td>
<td>&lt; 70/360=0.19</td>
<td>-20</td>
</tr>
<tr>
<td>LH</td>
<td>&gt; 0.19</td>
<td>160</td>
</tr>
<tr>
<td>LH</td>
<td>&gt; 0.69</td>
<td>340</td>
</tr>
<tr>
<td>LH</td>
<td>&gt; 1.2</td>
<td>520</td>
</tr>
<tr>
<td>RH</td>
<td>&lt; 110/360=0.31</td>
<td>-20</td>
</tr>
<tr>
<td>RH</td>
<td>&gt; 0.31</td>
<td>-200</td>
</tr>
<tr>
<td>RH</td>
<td>&gt; 0.81</td>
<td>-360</td>
</tr>
</tbody>
</table>
Substrates with alignment layers are ready for assembly in the cleanroom. First, both substrates are cleaned thoroughly with high pressure air gun and examined with black light for any traces of particles. Small particles left behind and imperfections at the edge of mirror stack are removed carefully with razor blade if necessary. After that very thin line of UV glue (Norland 65) is applied on one of the substrates using the computerized glue dispenser (see Figure 4A.4).

![Figure 4A.4: Diagram of glue lines and the fill port of the substrate prior to assembly](image)

The substrate with glue line is put on the vacuum plate and the other substrate is placed on top of it before covering them with plastic wrap for vacuum suction. Under the vacuum suction two substrates are pulled together and the UV glue spreads until the substrates are in contact with each other through the SiOx post spacers. This can be confirmed when interference fringes under monochromatic light source stops moving and relatively large (5mmX5mm) area of uniform thickness is achieved. Even pressure with
metal spatula can be applied along the regions with glue in order to accelerate this process which can take up to 30min. Finally, UV light source is placed on top of the vacuum mat and the cell is baked for 15min.

When the cell is cured with UV glue, the thickness is determined by measuring the transmittance of white light using spectrometer and fitting the data with numerical model. After that electric wires are attached to the cell and the vacuum filling process begins. Figure 4A.5 shows the picture of filling station where LC mixture in vial and empty cell are placed inside the vacuum chamber. Usually the chamber is pumped down to ~30micronHg and small amount of LC material is applied at the fill port of the cell using metal spatula.

![Figure 4A.5: Picture of the vacuum filling station](image)
Once fill port is covered with material the chamber is slowly vented so that LC mixture starts filling the empty cell. In order to get uniform twist alignment it's important to fill the cell while LC is in isotropic phase. To do so we remove the cell from the vacuum chamber after venting, and then place it on preheated hot plate (~110°C) immediately (See Figure 4A.6). In the isotropic phase LC fills faster and it's important to seal the fill port with epoxy glue before it completely fills the cell. Usually premixed 5min epoxy is applied when the cell is 2/3 full so that slight vacuum is left within the cell.

Figure 4A.6: LC is filled in the isotropic phase by placing the cell on a preheated hot plate after the vacuum
4.10 Appendix 4B - Reflectance/Transmittance Measurements

Sketch of the reflectance and transmittance measurements is shown in Figure 4B.1. For light source we used Oriel fiber optic illuminator (model 77501) and for spectrometer OceanOptics spectrometer (LS-1) was used. Note that in the setup path length of the transmitted and reflected lights are the same.

Figure 4B.1: Experimental setup for spectrometer measurements

Figure 4B.2 shows the actual experimental setup of the measurement station. Spectrometer was moved to two different positions for reflective and transmissive measurements (transmissive setup is shown).
Using this setup we measure the transmittance and reflectance of LC etalon cell and fit the data with computer model for analysis. Note that in all reflectance and transmittance spectrums 1 corresponds to the intensity measured with nothing in-between the light source and the spectrometer. This means that everything measured is normalized by the total incident unpolarized light intensity.
Chapter 5

Reflective Liquid Crystal Etalon Device

5.1 Introduction

5.1.1 Need for High Efficiency Reflective Display Device

With the rapid commercialization of portable electronic devices such as portable media players, cell phones, personal digital assistants, mobile internet devices, netbook and notebook computers, the need for sunlight readable power efficient displays is becoming ever more important. Constant improvements in contrast, color saturation, and viewing angle properties made thin-film transistor (TFT) liquid crystal (LC) display the dominant technology in both portable and large screen directview display applications. However, polarization dependent operation of LC displays combined with side-by-side color pixel structure makes the current technology very inefficient for use in battery operated portable devices. Current devices lose more than ~50% of the light due to absorption of polarizers and lose additional ~67% due to the color filters in the side-by-side sub-pixel structure.
5.1.2 New Reflective Display Devices

Due to the inherent inefficiency of current TFT LC display devices completely new technologies are emerging such as the iMoD displays from Qualcomm and the electrophoretic displays from E-Ink. Although these technologies cannot replace the existing technology in large area plugged-in display devices, they have very low power consumption and thus are well suited for niche applications like low end cell phones and electronic paper readers.

Etalons are usually considered as narrow bandwidth transmission filters whose transmission is not very high. But they can be designed for broader bandwidth, high transmission applications, and can also be considered as high reflectivity pixels for color displays. The idea is that when the etalon is in non-resonant mode, light for the spectrum of the mirrors is reflected to the viewer, but when the etalon is in resonant mode for the wavelength spectrum of the mirrors, then the light of that spectrum is transmitted, and the reflectivity is low (See Figure 5.1).

![Reflective etalon operation](image)

Figure 5.1: Reflective etalon operation
iMoD display from Qualcomm works in the same way as Fabry-Perot interferometer or an etalon such that the reflected light is tuned by mechanically changing the spacing between parallel mirrors. It has high brightness and low power consumption. Due to the mechanical design of etalon, these displays work in binary mode and pulse width modulation is needed for gray scale production. Obvious advantage of such design is that optical path length, $nd$, can be tuned to any value. For example, cavity spacing for iMoD device is tuned from 50nm to 400nm in order to produce either black or red reflection. Since cavity is filled with air, optical path length in this case has been varied by as much as 350nm.

5.1.3 LC Etalon for Reflective Display Application

In a LC based etalon, unlike conventional LC displays, tunability is achieved directly by phase retardation rather than polarization rotation effect and as a result it can be made polarization insensitive (See Figure 5.2). However, we cannot tune optical path length by large amount in liquid crystal etalon device for low order devices. In fact, the largest birefringence, $\Delta n$, of commercially available liquid crystals is not much bigger than 0.25. If we construct etalon with cavity thickness around 0.5µm, then available tunability of the optical path length, $\Delta nd$, would be only $\sim$125nm which is not enough for full color tunability in the visible spectrum.
Here we will explore a LC etalon device that employs low loss dielectric mirrors and high birefringence nematic LC mixture for switchable color pixels in reflective display application. We have considered different approaches to design a tunable LC etalon device. First, design of a simple quarter wave (\(\lambda/4\)) stack with an LC layer in the middle is considered. We calculated relative brightness and contrast ratio for 30nm wide green spectrum and investigated ways to optimize them. We then propose a \(\lambda/4\) stack mirror design with integrated half wave (\(\lambda/2\)) layers and LC layer. The contrast ratio of this design is significantly better. Finally, a modification that allows for a polarization independent operation is demonstrated.

5.2 Device Design

5.2.1 Analytical Calculations

In this section we will introduce the basics of tunable parallel-plate etalon device with the cavity filled with LC material. In a simple parallel-plate etalon incident light
undergoes multiple reflections within the cavity and interferes with itself (See Figure 5.3).

\[ T = \frac{T_a T_b}{1 - \sqrt{R_a R_b}} \left[ 1 + \frac{4 \sqrt{R_a R_b}}{(1 - \sqrt{R_a R_b})} \sin^2 \left( \frac{\phi_a + \phi_b}{2} - \delta \right) \right]^{-1} \]  

Eq. (5.1)

Here, \( T_{a,b} \), \( R_{a,b} \) are transmittance and reflectance of parallel mirrors, \( \phi_{a,b} \) is phase shift on reflection from the mirrors and \( \delta = (2 \pi d/\lambda) \). If the absorption is negligible, then reflectance of the etalon is simply \( R = 1 - T \), which has minimums and maximums under conditions given by Eq. (5.2).

\[ (nd)_{\text{dark}} = \frac{\lambda_0}{2} m, \quad (nd)_{\text{bright}} = \frac{\lambda_0}{2} \left( m \pm \frac{1}{2} \right) \]  

where, \( m = 1,2,3... \)  

Eq. (5.2)
Optical path length of the cavity, \( nd \), is the primary variable by which the etalon reflectance is tuned. In a mechanical etalon device tunability is achieved by changing the physical thickness of the cavity, whereas in an LC etalon device reflectance is tuned by switching effective index of the cavity by an applied electric field. A nematic LC is characterized by an orientational ordering which is defined by unit vector (the director) along the average direction of elongated molecules’ long axis. Light polarized along and perpendicular to the director axis experiences different indices of refraction which are called the extraordinary and ordinary LC indices.

Using Eq. (5.2) if one chooses ordinary refractive index, \( n_o \), to correspond to the dark state, then extraordinary index, \( n_e \), will correspond to the bright state with +1/2. However, if one chooses \( n_e \) to be the dark state than \( n_o \) must be the bright state with -1/2. These two cases are presented in Table 5.1 shown below. For each case the required index for the maximum brightness of the bright state is given.

Table 5.1: \( \lambda=535\text{nm} \)
It can be seen that for values of $m = 1$-3, the calculated value of the index for the bright state is outside of the obtainable range. The 4th and 5th order devices (in bold) optimize the full range of refractive indices that one can tune for our hypothetical liquid crystal with $n_o=1.5$ and $n_e=1.75$. This gives us an idea about what thickness of the cell we will be considering for optimum brightness and contrast ratio.

Using the Eq. (5.1) and (5.2) we can calculate contrast ratio and average reflectivity for select wavelength region to see the approximate effects of thickness, and mirror reflectivities.

<table>
<thead>
<tr>
<th>Table 5.2: ($520\text{nm} &lt; \lambda &lt; 550\text{nm}$)</th>
<th>Dark State</th>
<th>Dark State</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_o=$</td>
<td>1.55</td>
<td>$n_e=$</td>
</tr>
<tr>
<td>$m$</td>
<td>$d$ (um)</td>
<td>$n_b$ (bright)</td>
</tr>
<tr>
<td>1</td>
<td>0.173</td>
<td>2.325</td>
</tr>
<tr>
<td>2</td>
<td>0.345</td>
<td>1.938</td>
</tr>
<tr>
<td>3</td>
<td>0.518</td>
<td>1.808</td>
</tr>
<tr>
<td>4</td>
<td>0.690</td>
<td>1.744</td>
</tr>
<tr>
<td>5</td>
<td>0.863</td>
<td>1.705</td>
</tr>
<tr>
<td>6</td>
<td>1.035</td>
<td>1.679</td>
</tr>
</tbody>
</table>
Note that green cells represent devices that have \( n_o \) as dark state and yellow cells represent devices that have \( n_e \) as dark state. For two green devices that have thicknesses greater than 0.764\( \mu \)m, dark states correspond to \( n_e \)'s less than 1.75. For the same reason yellow device with thickness 0.917mm has bright state with \( n_o \) more than 1.55. One can see that as thickness of the cavity decreases the contrast ratio goes up. On the other hand high reflectivity of mirrors results in better brightness. Therefore, devices that have good contrast ratio and high brightness are in the upper right side of Table 5.2.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( r^2 = 0.15 )</td>
<td>( r^2 = 0.20 )</td>
<td>( r^2 = 0.25 )</td>
<td>( r^2 = 0.30 )</td>
<td>( r^2 = 0.50 )</td>
<td>( r^2 = 0.70 )</td>
</tr>
<tr>
<td>d  [( \mu )m]</td>
<td>CR</td>
<td>Brightness</td>
<td>CR</td>
<td>Brightness</td>
<td>CR</td>
<td>Brightness</td>
</tr>
<tr>
<td>1</td>
<td>0.150</td>
<td>43.1</td>
<td>0.09</td>
<td>41.1</td>
<td>0.13</td>
<td>39.1</td>
</tr>
<tr>
<td>2</td>
<td>0.170</td>
<td>54.1</td>
<td>0.12</td>
<td>51.1</td>
<td>0.17</td>
<td>47.1</td>
</tr>
<tr>
<td>3</td>
<td>0.200</td>
<td>31.1</td>
<td>0.26</td>
<td>27.1</td>
<td>0.34</td>
<td>24.1</td>
</tr>
<tr>
<td>4</td>
<td>0.345</td>
<td>35.1</td>
<td>0.80</td>
<td>31.1</td>
<td>0.39</td>
<td>27.1</td>
</tr>
<tr>
<td>5</td>
<td>0.493</td>
<td>21.1</td>
<td>0.88</td>
<td>18.1</td>
<td>0.48</td>
<td>16.1</td>
</tr>
<tr>
<td>6</td>
<td>0.518</td>
<td>22.1</td>
<td>0.42</td>
<td>19.1</td>
<td>0.52</td>
<td>16.1</td>
</tr>
<tr>
<td>7</td>
<td>0.611</td>
<td>14.1</td>
<td>0.44</td>
<td>12.1</td>
<td>0.54</td>
<td>10.1</td>
</tr>
<tr>
<td>8</td>
<td>0.630</td>
<td>14.1</td>
<td>0.44</td>
<td>12.1</td>
<td>0.54</td>
<td>10.1</td>
</tr>
<tr>
<td>9</td>
<td>0.764</td>
<td>9.1</td>
<td>0.44</td>
<td>8.1</td>
<td>0.54</td>
<td>7.1</td>
</tr>
<tr>
<td>10</td>
<td>0.808</td>
<td>9.1</td>
<td>0.44</td>
<td>8.1</td>
<td>0.54</td>
<td>7.1</td>
</tr>
<tr>
<td>11</td>
<td>0.917</td>
<td>7.1</td>
<td>0.43</td>
<td>6.1</td>
<td>0.54</td>
<td>5.1</td>
</tr>
<tr>
<td>12</td>
<td>1.038</td>
<td>7.1</td>
<td>0.43</td>
<td>6.1</td>
<td>0.53</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Consider device A-8 on Table 2. The contrast ratio and the brightness of the device are 14 and 0.44 respectively. Let us look at other devices that have similar contrast ratio but better brightness. In order for the brightness to increase one has to increase the reflectivity of the mirrors. As a result brightness of devices increase as one goes to the right in row 8. However, contrast ratio decreases as one goes to right of device A-8. In fact the brightest device in row 8, device F-8 has the worst contrast ratio. Consequently
one has to decrease the thickness of the device in order to keep the contrast ratio same as device A-8. Devices B-7 and D-5 have similar contrast ratio the as device A-8 but better brightness.

Now consider devices that have better contrast ratio but similar brightness as device A-8. All devices in column A that are above A-8 (devices A-1 to A-7) have better contrast ratio because of the small cavity thickness. However, brightness decreases as one goes up in column A. Device A-1 has the best contrast ratio but has brightness of only 8%. As a result one has to go to upper right corner of Table 5.2 for better contrast ratio without sacrificing the brightness. For example, devices B-4 and C-3 have better contrast ratio even when their brightness is comparable to that of device A-8. From this approximate model we can see that we will need to consider very thin devices (< 0.5 microns).

It can also be seen from Eq. (5.2) that the ratio between effective indices of the dark and bright states reduces as the order of the device gets higher \((n_{\text{dark}} / n_{\text{bright}} = m / (m - 1/2) \quad m = 1,2,3,\ldots)\). This means that if the LC index is to switch from first order (m=1) dark state to first order bright state then the required change in index is much larger than if it is to switch between higher order dark and bright states. It should be noted here, however, that the condition for high reflectivity is valid only at the center wavelength, \(\lambda_0\), so for other wavelengths the condition may not be met. This wavelength dispersion effect of reflectance gets severe as the order of the device increases and as a result lowest possible order device which requires high birefringence LC material is
needed. Although state of the art materials with birefringence as high as 0.6 are reported\textsuperscript{63}, current commercially available LCs have birefringence not more than around 0.3\textsuperscript{62}.

5.2.2 Dielectric Mirror Design and Simple Quarter-wave Stack

In this section we will first briefly introduce the dielectric mirror design considerations. Specifically high reflectivity caused by stack of alternating quarter-wave layers where the refractive index modulation resembling the square wave profile will be covered. As it will become clear such square wave index modulation results in higher harmonics manifested by high reflectivity regions in the spectrum that is different than the wavelength at which the layers are quarter-wave. We will then cover the method to eliminate the presence of higher harmonics by sine wave index modulation and will also explain how to reduce the reflection side lobes by apodizing the index profile.

Notch filters made with discrete $1/4\lambda$ layers of alternating high and low index materials exhibit high reflection around the center wavelength and high transmission far from the center wavelength. The wavelength reflective properties of alternating high and low index quarter-wave stack and the spatial distribution of the refractive index profile are Fourier transform pair\textsuperscript{86,87}. As a result filter made with discrete square-wave profile of refractive index suffers from the presence of harmonic structure. Simple frequency domain analysis of square wave is a good illustration for this. Square wave with peak-to-peak amplitude of unity can be expressed in Fourier-series as:
\[ \frac{4}{\pi} \left( \frac{\sin x}{1} + \frac{\sin 3x}{1} + \frac{\sin 5x}{1} + \ldots \right) \]  

Eq. (5.3)

Figure 5.4 shows the first 9 components of Fourier series of a square wave. Fourier transform of this series in frequency domain shows distinct peaks at odd frequency interval (see Figure 5.5) which is a characteristic of notch filter made of square-wave index profile. Figure 5.6 shows the refractive index profile of a notch filter with an average index 1.75, and peak to peak amplitude 0.25. Optical response of this notch filter is shown in Figure 5.7 which has a large rejection band at the center wavelength, 525nm, and at its odd multiples, 175nm, 105nm etc. Note that each parameter of the square-wave: amplitude, period, average index, and the number of period influences different properties of the notch filter. The amplitude of the wave determines the width of the
notch, the number of period determines the depth of the notch, and the period determines the notch wavelength. Figure 5.8 and Figure 5.9 show the sine-wave refractive index profile of a notch filter and its optical response. Note that harmonic rejection bands have disappeared, because higher frequency components of the square-wave were excluded from the index profile. Since the peak-to-peak amplitude in sine-wave index profile is same as that of the square-wave profile, rejection bandwidth of sine-wave filters is $\pi/4$ times that of the square-wave filters.

Figure 5.5: Fourier transform in the frequency domain
Figure 5.6: Square-wave profile of refractive index with period of $1/2\lambda$ at 525nm

Figure 5.7: Optical response of a notch filter with square-wave index profile
Figure 5.8: Sine-wave profile of refractive index with period of $1/2\lambda$ at 525nm

Figure 5.9: Optical response of a notch filter with sine-wave index profile

For a switchable high efficiency reflective device the simplest design is to use dielectric mirrors that consist of stack of alternating high and low index material. In this
design LC layer in the etalon cavity (between two mirrors) switches between $\lambda/4$ and $\lambda/2$ states to get high and low reflectances. When the LC layer is in $\lambda/4$ state constructive interference of reflections from each dielectric interface result in reflectance maximum, and when it switches to $\lambda/2$ state reflection from top half of the mirror destructively interferes with that from bottom half and reflectance minimum occurs. However, reflectance rises as the wavelength diverges away from the center wavelength where the dielectric stack is tuned. Such light leakage outside the center wavelength but within the operating spectrum decreases the contrast of the device.

Although the sinusoidal index profile eliminates the higher harmonic rejection bands it doesn't decrease the amplitude of the side-lobes. Apodization of the refractive index profile is needed for decreasing the side-lobes' amplitude. The refractive index profile as a function of optical thickness, $x$, is shown below.

$$n(x) = n_a + \frac{1}{2}n_p A(x) \sin\left(\frac{4\pi x}{\lambda}\right)$$  \hspace{1cm} \text{Eq. (5.4)}

Here, $n_a$ and $n_p$ are the average index and the peak to peak index amplitude respectively, and $A(x)$ is the apodization function. The apodization function is a polynomial and is a function of optical thickness as well. Previously it was found by Southwell et al\textsuperscript{88} that having a fifth order polynomial as an apodization function was optimum in suppressing the side-lobes completely.

$$A(x) = 10r^3 - 15r^4 + 6r^5$$  \hspace{1cm} \text{Eq. (5.5)}
Where, \( t = 2x/T \) for \( x < T/2 \), and \( t = 2(T - x)/T \) for \( x > T/2 \), with \( T \) being the total optical thickness. Figure 5.10 and Figure 5.11 show a sample apodized index profile with half-wave layer in the middle and the corresponding reflectance spectrum.

Figure 5.10: Optical structure of apodized sinusoidal profile with inserted half-wave in the middle
5.2.3 Integrated Half-wave Layers Design

In order to increase the contrast of the device light leakage outside the center wavelength in the dark state must be suppressed. For broader dark state it is possible to couple several \( \lambda/4 \) stacks separated by \( \lambda/2 \) layers in order to get multiple reflection minimums. This is referred to as a multiple cavity filter. For example, Figure 5.12 shows a simple series in which three reflector stacks consisting of alternating \( \lambda/4 \) layers (in dark) conjoined by two \( \lambda/2 \) layers (in white). It is intuitive to think of the total reflection from the series as the sum of reflections from individual parts. Then reflection from the sub-series which consist of top two reflectors sandwiching the first \( \lambda/2 \) layer has minimum at the center wavelength (V shaped reflection) whereas reflection from the bottom reflector has maximum at the center wavelength. Total reflection of the series is the difference between reflections from the sub-series and the bottom reflector, because
they are conjoined by the second \( \lambda/2 \) layer and are out of phase. If the amplitudes of these reflections are comparable, then there will be two minimums in the total reflection at the intersection points where these reflections are equal (W shaped reflection). Similarly if third \( \lambda/2 \) layer and another reflector consisting of \( \lambda/4 \) stack are added in the series, then the resultant total reflectance would be the difference between W shaped reflection coming from the subseries which consist of three reflectors and two \( \lambda/2 \) layers, (the stack of Figure 5.12) plus the reflection from the fourth reflector. In this case it is not hard to imagine that depending on the reflection amplitudes it is possible to have up to four minimums in the total reflection.

Figure 5.12 Left: graphical representation of the reflection from two \( \lambda/2 \) layers in series with three \( \lambda/4 \) stacks; Right: total reflectance corresponds to the stack: QAQAQ; reflectance 1 stack is: QAQ; and the reflectance 2 stack is: Q; where, \( Q=n_Hn_Ln_H \), \( A=n_Ln_L \), \( n_H=1.45 \), \( n_L=1.38 \), \( d_H=90.5\text{nm} \), \( d_L=95.1\text{nm} \)
In our proposed design we integrated two $\lambda/2$ layers within the dielectric mirrors and tuned the LC cavity between $\lambda/2$ and $\lambda/4$ states (See Figure 5.13 and Figure 5.14). When the LC is in the $\lambda/2$ state, the etalon resembles the series with three $\lambda/2$ layers (Figure 5.13), whereas when the LC is in $\lambda/4$ state, the etalon becomes a series with two $\lambda/2$ layers (Figure 5.14). The main goal of the design is to achieve broadband dark state with minimum light leakage when the LC is in the $\lambda/2$ state and to maximize the bright state reflection when the LC switches to $\lambda/4$ state. In order to increase the bandwidth of the dark state, total reflectance minimums where reflectance 1 and 2 intersect must be far apart (right plot in Figure 5.13). This means the width of W shaped reflection from the sub-series which consist of two $\lambda/2$ layers and three reflector stacks must be increased. This in turn requires the width of V shaped reflection from the sub-sub-series which consist of single $\lambda/2$ layer and two reflector stacks to be wide (the “reflectance 1” curve in Figure 5.12). There are two ways to increase the width of this reflection which are to increase the ratio of high and low index materials, or to decrease the number of layers in the $\lambda/4$ stack. In addition to increasing the dark state bandwidth, it is also important to keep the light leakage in the dark state as low as possible which means the amplitude of reflections from the sub-series (W shaped reflection) and the bottom reflector stack must be comparable near their intersection points. In general this is done by keeping the reflections of two inner $\lambda/4$ stack reflectors comparable to those of two outer $\lambda/4$ stack reflectors which in effect reduce the difference between reflectance 1 and 2 within their
intersection points (right plot in Figure 5.13). As for the bright state, the reflection would be maximized if all of the $\lambda/2$ layers switched to $\lambda/4$. However if only the middle $\lambda/2$ layer (the LC etalon) switches to $\lambda/4$, then the series becomes similar to that shown in Figure 5.12. In this case however, the reflectance of the middle $\lambda/4$ stack reflector is much higher than that of the outside $\lambda/4$ stack reflectors and reflections from two parts of the series do not intersect (right plot in Figure 5.14). As a result total reflectance of the series has slight dip at the center wavelength without touching zero.

Figure 5.13 Left: graphical representation of the reflection from three $\lambda/2$ layers in series with four $\lambda/4$ stacks; Right: total reflectance corresponds to the stack: QAQ₂AQ₂AQ; reflectance 1 stack is: QAQ₂AQ₂; reflectance 2 stack is: Q; where, $Q=n_H n_L n_H$, $Q_2=n_H n_L n_H n_L n_H$, $A=n_L n_L$, $n_H=2.20$, $n_L=1.38$, $d_H=59.7\text{nm}$, $d_L=95.1\text{nm}$
Figure 5.14 Left: graphical representation of the reflection from two $\lambda/2$ layers in series with three $\lambda/4$ stacks; Right: total reflectance corresponds to stack: QAQ$_2$AQ; reflectance 1 stack is: Q; reflectance 2 stack is: QAQ$_2$; where, $Q=n_H n_L n_H$, $Q_2=(n_H n_L)^5 n_H$, $A=n_L n_L$, $n_H=2.20$, $n_L=1.38$, $d_H=59.7$nm, $d_L=95.1$nm

In summary, the dark state bandwidth of a $\lambda/4$ dielectric stack with triple $\lambda/2$ layers increases with increasing ratio of high and low index materials and with lowering of reflectivities of the two inner $\lambda/4$ stack reflectors. However, reflectivities of the two inner $\lambda/4$ stack reflectors must be comparable to those of outer $\lambda/4$ stack reflectors for minimum light leakage. If all reflectivities are reduced, then the reflectance in the bright state is also reduced. As a result the bandwidth and the light leakage of the dark state and the reflection of the bright state must all be balanced according to the specific application requirements.
In these etalon designs the theoretical reflectance is limited to less than 50% of an unpolarized incident light, because polarization sensitive LC operation requires a polarizer that absorbs more than half of the incident light. If the tunable LC layer within the etalon cavity is made polarization insensitive, then the etalon brightness will be enhanced by two fold. A method to achieve polarization insensitive transmissive LC etalon operation has been published by the authors. In this paper we extend the method to reflective LC etalon operation.

It was shown in Eq. (5.2) that the reflectance is maximized when \((nd)_{LC}\) is an odd multiple of a quarter-wave. In a homogeneous non-twist LC medium, the eigenmodes of light propagation are linearly polarized modes corresponding to LC ordinary and extraordinary indices. The total reflectance becomes the sum of these modes, only one of which is tunable. Previously it was shown by the authors that polarization independent LC etalon operation can be achieved if the phase shift on reflection from the mirrors and the phase conditions for etalon resonance for the two eigenmodes are tuned by having twist LC structure. Instead of reducing the phase difference between the eigenmodes, polarization independent LC etalon operation is achieved by simultaneously satisfying different orders of resonances for the elliptically polarized eigenmodes. Phase shift on reflection from the dielectric mirrors required the difference in the phases of the eigenmodes to be integer multiple of \(2\pi\). For reflective LC etalon, eigenmodes of the light propagation must acquire different orders of resonance condition where \((nd)_1=\lambda m/2\) and \((nd)_2=\lambda n/2\) with \(m=1,3,5\) and \(n=1,3,5\) but \(n\neq m\). This means the difference between \(m\) and \(n\) is an even number and the condition arising from the phase shift on reflection from
the dielectric mirrors is always satisfied. Once the mirror reflectivity, LC cavity thickness and material parameters are known, the LC twist angle that reduces the polarization sensitivity will be found and be implemented in the design.

In our design we will optimize the etalon mirrors for green wavelength region ranging from 510nm to 540nm. In the next section we will numerically calculate reflectance of different designs of LC etalon and will analyze them in terms of brightness and contrast.

5.3 Numerical Model

5.3.1 Quarter-wave Stack Design

In this section we will numerically simulate two different LC etalon designs for green wavelength region from 510nm to 540nm. Optical reflectance of the LC etalon was calculated by dividing the entire structure into finite number of layers of individual optical elements. Numerical method that we used was 4x4 matrix formulation which is a direct solve of Maxwell's equations in one dimension. First we consider a polarization dependent LC etalon where dielectric mirrors consist of stack of alternating high and low index $\lambda/4$ materials. Then we consider our proposed design with integrated $\lambda/2$ layers which has wider dark state and better contrast. For these designs homogeneous LC structure was modeled for polarized light source as dielectric medium with constant refractive indices and total reflectance is halved for polarization loss. After introducing these designs we will modify the design to be polarization independent. In this case twist
LC director structure was calculated by minimizing the free energy which consists of Frank-Oseen elastic energy and electric energy due to applied field. Material parameters of the modeled LC were same as that of BL009 from Merck ($\varepsilon//=21$, $\varepsilon\perp=5.5$, $\gamma\sim0.083$PaS, $K_{11}\sim14.6$pN, $K_{22}\sim7$pN, $K_{33}\sim29.9$pN) which has birefringence near 0.29. We modify the birefringence and keep other material parameters the same for high birefringence designs. We ignore the effects of glass substrates, conducting and alignment layers in the optical calculations, because they can be minimized and or compensated in real devices. Calculated reflectances were normalized by incident unpolarized light.

It is preferable to have a low order etalon device in order to minimize the wavelength dispersion effects on reflectance. A typical nematic LC material has ordinary index of refraction around 1.53, and if we choose to make this index correspond to dark state, then thickness of the 1\textsuperscript{st} order device for 525nm wavelength is calculated using Eq. (5.2) to be $\sim0.172\mu$m. Using the same equation we can calculate the required extraordinary index of the LC to tune from 1\textsuperscript{st} order dark state to 1\textsuperscript{st} order bright state to be around $\sim2.29$. This is birefringence in excess of 0.7 which is not practical and as a result order of the device is increased to reduce the required birefringence. It can be further calculated using Eq. (5.2) that the required birefringence for 2\textsuperscript{nd} order device is $\sim0.4$ and that for 3\textsuperscript{rd} order device is $\sim0.3$. In our design etalon mirrors are modeled as stack of alternating layers of high and low index ($n_h\sim2.27$, $n_l\sim1.63$) dielectric materials tuned to be $\lambda/4$ at 525nm. For enhanced reflectivity we simply increased the ratio between high and low indices of the dielectric materials.
Calculated reflectance spectrum of unpolarized incident light for 1\textsuperscript{st}-3\textsuperscript{rd} order LC etalon devices are shown in Figure 5.15. It shows that low order device has better contrast as a result of small light leakage in the dark state. In addition low order device has higher reflectance in the bright state because of the increased index mismatch at the interface of dielectric mirror and LC layer. Note that the dielectric mirrors were designed only for 30nm wide green region of the visible spectrum and LC etalon reflectance is optimized in this region only. The inset in Figure 5.15 shows reflections outside the optimized region (in grey shade). Figure 5.16 shows the reflectance spectrum of LC etalon with high reflectivity mirrors. In this design mirror reflectivity is enhanced by increasing the ratio between high and low index materials. In comparison to the previous design the reflectance of the etalon in the bright state is increased considerably. However, light leakage outside the center wavelength region in the dark state is elevated and as a result contrast of the device is not so high.
Figure 5.15: Reflectance spectrum of 1-3rd order LC etalon device. Optical structure of the etalon is: \((n_L n_H)^2 n_L n_{LC} n_L (n_H n_L)^2\) where \(n_H = 2.27\), \(n_L = 1.63\)

![Reflectance spectrum of 1-3rd order LC etalon device](image)

Figure 5.16: Reflectance spectrum of 1-3rd order LC etalon device. Optical structure of the etalon is: \((n_L n_H)^2 n_L n_{LC} n_L (n_H n_L)^2\) where \(n_H = 2.27\), \(n_L = 1.45\)

5.3.2 Integrated Half-Wave Stack Design

It was mentioned in the previous section that etalon with single \(\lambda/2\) layer sandwiched between two \(\lambda/4\) reflector stacks as shown in Figure 5.17 and has low contrast, because of the reflection rise outside the center wavelength. If additional \(\lambda/2\) layers and reflector stacks are inserted within the etalon, then the contrast is improved as a result of wider dark state bandwidth. With the constraints introduced in the previous section we have modeled LC etalon with 2 integrated \(\lambda/2\) layers in the dielectric mirrors.
Index ratio was maximized \( (n_H \sim 2.27, n_L \sim 1.38) \) for broad dark state at the same time maximizing the bright state reflectivity. Number of layers in the inner and outer \( \lambda/4 \) stacks was optimized so that reflectance in the bright state is maximized at the same time keeping the light leakage in the dark state minimized. Figure 5.19 shows reflectance spectrum of unpolarized incident light on 3\(^{rd}\) order etalon device with integrated half-wave layers. Design IIA (solid curve) has considerable light leakage in the dark state that goes up to 10\% along the edges of the spectrum. Dashed curve shows increased dark state bandwidth as a result of reduction in the reflectivity of the inner \( \lambda/4 \) stacks. This is done by changing the indices of the two dielectric materials next the LC layer (\( \ldots n_H n_{LC} n_H \ldots \)) from 2.27 to 1.95. However, reduced inner \( \lambda/4 \) stack reflectivity results in drop in the bright state reflectance at the center wavelength region. Similarly reflectivity of the outer \( \lambda/4 \) stack can be reduced by lowering the index of the outermost high index materials from 2.27 down to 1.95. In this case dark state bandwidth is narrowed and the reflectance in the bright state is higher (dotted curve). As with the previous design reflections outside 30nm wide green region must be suppressed (shaded area in the inset of Figure 5.19) with the help of additional absorptive color filters in a display device.
Figure 5.17: Optical structure of the design is:

\[(n_Ln_H)^2n_L^2n_H^2n_Ln_HCn_H(n_Ln_H)^2n_Ln_H^2n_Ln_HCn_H^2\]

where \(n_H = 2.27\), \(n_L = 1.38\), \(n_{LC} = 1.53/1.78\).

All dielectric layers are \(\lambda/4\) thick at 525nm and LC layer thickness is 0.515\(\mu\)m. Please see text for description of stacks with reduced reflectivity.

It is interesting to see how the order of the etalon affects the reflectance spectrum in integrated half-wave design. Figure 5.18 shows reflectance spectrum of the same LC etalon that is shown in Figure 5.17 except this time LC layer thickness is decreased by one order. As with the \(\lambda/4\) stack mirrored etalon design integrated half-wave etalon design has broader dark state as the order of the device decreases. At the same time however, light leakage is slightly increased just outside the 525nm region and the average reflectance is lowered in the bright state.
Figure 5.18: Optical structure of the design is:

\[(n_L n_H)^2 n_L n_L(n_H n_L)^2 n_H n_L n_H(n_L n_H)^2 n_L n_L(n_H n_L)^2\]

where \(n_H = 2.27\), \(n_L = 1.38\), \(n_{LC} = 1.53/1.91\).

All dielectric layers are \(\lambda/4\) thick at 525nm and LC layer thickness is 0.343\(\mu m\).

### 5.3.3 Polarization Independent Reflective LC Etalon

For high efficiency, as mentioned in the introduction, we consider a twisted LC device to yield a polarization independent device. As the LC twist angle increases, the polarization of eigenmodes changes from pure linear modes to elliptically polarized modes and the effective indices approach the average of ordinary and extraordinary indices. This means that the effective index in the field off state decreases with high twist rate, whereas that in the field on state stays the same. As a result switching between \(\lambda/2\) and \(\lambda/4\) states requires higher birefringence material in twisted LC etalon than in non-twist LC etalon. In the quarter-wave stack LC etalon design, 1\(^{st}\) and 2\(^{nd}\) order etalons require birefringence \(~0.4\) and \(~0.7\) for a non-twisted LC layer (see Figure 5.15). A
twisted configuration would require even higher value of birefringence which is not practical. However, the 3rd order device with integrated half-wave stack design shown in Figure 5.17 requires birefringence of only ~0.25. For this device we need to find the twist angle that satisfies the resonance condition which is consistent with the requirements of the phase shift on reflection from the mirrors. Using equation 6 in the reference paper\(^{58}\) we have calculated the relative phase difference between the eigenmodes as function of twist angle (left plot in Figure 5.19) and found the lowest twist angle at which the phase difference is integer multiple of \(2\pi\) (\(4\pi\) at \(\sim 170^\circ\)). We then calculated the acquired phases of the eigenmodes at this twist angle (right plot in Figure 5.19) using equation 5 in the reference paper\(^{58}\) and found them to be \(4.4\pi\) and \(2.4\pi\). They are not odd multiple of \(\pi/4\) and as a result resonance condition for reflectance maximum is not fully satisfied. However, if the material birefringence is 0.25, then at this twist angle LC etalon is least polarization sensitive for the wavelength region near 525nm. To confirm our result we calculated the average reflectance for different twist angle and plotted in Figure 5.20. It shows that the optimum twist angle at which the reflectance is maximum is between \(150^\circ\) and \(180^\circ\) which is consistent with our predictions. Although reflectance is increased significantly compared to design IIA in Figure 5.17, it is not twice as high as the polarization sensitive reflectance. This is due to the fact that effective birefringence of the twist LC structure is much lower than the non-twist LC structure. As a result higher birefringence LC material is needed in order to further increase the reflectance of the polarization insensitive etalon. Figure 5.21 shows the reflectance of polarization insensitive etalon where the birefringence of the material is 0.3 and 0.4.
Figure 5.19: $n_{LC}=1.53/1.78$, $d_{LC}=0.515\mu m$.

Figure 5.20: Optical structure of the etalon is:

$$(n_{L}n_{H})^2 n_{L}n_{L}(n_{H}n_{L})^2 n_{H}n_{L}n_{H}n_{L}(n_{H}n_{L})^2 n_{L}n_{L}(n_{H}n_{L})^2$$

where $n_{H}=2.27$, $n_{L}=1.38$, $n_{LC}=1.53/1.78$.

All dielectric layers are $\lambda/4$ thick at 525nm and LC layer thickness is 0.515$\mu m$. 
Figure 5.21: Optical structure of the etalon is:

\[(n_hn_l)^2n_l(n_hn_l)^2n_hn_{LC}n_h(n_hn_l)^2n_l(n_hn_l)^2\]

where \(n_h=2.27\), \(n_l=1.38\), \(n_{LC}=1.53/1.83\) (blue), 1.93 (red). All dielectric layers are \(\lambda/4\) thick at 525nm and LC layer thickness is 0.515\(\mu\)m.

### 5.4 Discussion

In the previous section, the numerical calculation results of a modeled etalon that operates over 30nm wide spectrum ranging from 510nm to 540nm were shown. We considered different designs for LC etalon device. A quarter wave stack design with single tunable LC layer has narrow dark state where light leakage outside the center wavelength was quite high. On the contrary \(\lambda/4\) stack with integrated multiple \(\lambda/2\) layers...
had much broader dark state even for high order devices that are required due to the limited birefringence of available LC materials.

Table 5.3 summarizes the results of numerical modeling for different LC etalon device designs. Integrated multiple half-wave design has higher reflectance and contrast compared to $\lambda/4$ stack mirror design. Even when the order of the etalon is low (design IB) reflectivity is not as high as the multiple $\lambda/2$ design (design IIA). Note that the design IIB has lower reflectance and contrast compared to design IIA even though it is lower order device. This is due to reduced reflectance as a result of lower index mismatch at the LC and high index dielectric interface, as well as elevated light leakage outside the center wavelength region in the dark state.

Table 5.3: Results of numerical modeling for different designs of LC etalon device (reflectance is normalized by the total incident un-polarized light). See figure captions for the design details.

<table>
<thead>
<tr>
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<th>Polarization Sensitive</th>
<th>Polarization Insensitive</th>
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<tbody>
<tr>
<td></td>
<td>Design IA</td>
<td>$\Delta n=0.25$</td>
</tr>
<tr>
<td>Operational Spectrum</td>
<td>30nm</td>
<td>30nm</td>
</tr>
<tr>
<td>Average Reflectance</td>
<td>25%</td>
<td>30%</td>
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<tr>
<td>Contrast Ratio</td>
<td>15</td>
<td>30</td>
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</table>
For polarization independent designs we optimized the twist angle that results in the highest reflectance in the integrated $\lambda/2$ design. Reduced effective birefringence due to twist LC structure requires higher LC material birefringence. This can be clearly seen in the Table 5.3 as birefringence of the LC approaches 0.4 reflectivity is double that of the polarization dependent design IIA.

In the designs of LC etalon device we did not discuss about unwanted reflections outside the spectrum bandwidth. The simplest solution would be to have cut-off color filters that absorb incident light outside the design spectrum. However, such design would force side-by-side color pixel structure as in conventional LC displays which would reduce the reflectance efficiency by more than 2/3. If dielectric mirrors are designed such that incident light outside the design bandwidth is transmitted, then it would result in stackable device where brightness efficiency would be near to those shown in Table 5.3 over the entire working wavelength regions (See Figure 5.22). Also the current active modulation spectrum width of 30nm for RGB pixels could be too narrow for bright white state, and therefore, it is possible to add an extra 30nm pixel to increase the overall reflectance such as shown in Figure 5.23.
In calculating the reflectance spectrum of above designs we did not consider the effects of alignment and conducting layers. However, it is possible to incorporate them into the design of the dielectric mirrors. Alignment and conducting layers can be substituted for either high or low index material by adjusting their optical thickness to be $\lambda/4$. Further optimizations can be done with precisely tailored dielectric mirror designs where index mismatch and order of the device better serve the overall brightness and contrast of the device.

Finally, we would like to comment on the effects of viewing angle and dielectric mirror dispersion as well as the order selection of the LC etalon on the performance of
the device. As Figure 5.13 shows the reflectivity of the device sharply increases outside the 30nm design bandwidth. This means that the contrast of the device is reduced for obliquely incident light as a result of increased light leakage coming from the change in apparent thickness of the dielectric stack. Figure 5.24 shows the comparison reflectances of design IIE for on-axis (solid) and for off-axis incident light (10° - dotted, and 15° - dashed). Numerical calculations indicate that the contrast of the design IIE drops down to about 30 for 10° off-axis light and to mere 10 for 15° off-axis light. Similarly, dispersion of high index dielectric materials can decrease the brightness and contrast of the device.

In order to illustrate this effect we modeled dielectric materials of the design IIE as Ta₂O₅, MgF₂, and high birefringence LC mixture of which indices are calculated using Cauchy equation, \( n = A + B/\lambda^2 + C/\lambda^4 \), with \( A_H = 2.16, B_H = -2.00 \times 10^3, C_H = 3.16 \times 10^9, A_L = 1.35; B_L = 4.07 \times 10^3, C_L = 0; \) and \( A_{\omega/c} = 1.44/1.76, B_{\omega/c} = 2.50 \times 10^4/3.50 \times 10^4, C_{\omega/c} = 0/3.00 \times 10^9 \). Dashed line in Figure 5.24 shows that the dispersion of the dielectric indices caused the dark and bright state reflectances to shift slightly to the longer wavelength region. In this case average reflectance and contrast of the design IIE is decreased down to \( \sim 70\% \) and 55 respectively. It was shown in § 2 that the lower order LC etalon has wider dark state bandwidth. As a result of this constraint LC cavity thickness in our design is thinner than in most of the commercially available products. However, while manufacturing of thin cells is difficult, the authors have shown elsewhere that the requirement for thickness non-uniformity for sub-micron cells is not necessarily severe. In addition, top-end manufacturers are currently producing cells that are 0.7 microns thick in high volumes and have stated that thinner cells are possible.
Figure 5.24: Optical structure of the etalon is:

$\frac{1}{2}(n_L n_H)^2 n_L n_L(n_H n_L)^2 n_H n_L n_H(n_L n_L)^2 n_L n_L(n_H n_L)^2$ where $n_H = 2.27$, $n_L = 1.38$, $n_{LC} = 1.53/1.93$ (blue), 1.93 (red). All dielectric layers are $\lambda/4$ thick at 525nm and LC layer thickness is 0.515 $\mu$m.

5.5 Summary

We have demonstrated LC etalon based switchable mirror which in one state reflects 30nm region of the visible spectrum and in the other state transmits it. Bandwidth of the operational spectrum and the amplitude of the reflection were optimized and inclusion of multiple half-wave passive layers found to have superior performance. In addition, polarization independent transmissive etalon operation reported previously by the authors has been extended for reflective device. For reflective etalon relative phase difference between the eigenmodes has to be integer multiple of $2\pi$ which satisfies both
the requirement for etalon reflectance maximum and the requirement for constructive interference of reflected light from the mirrors.

Numerical model demonstrates that polarization independent reflective device can be designed with reflectance ~70% and with contrast ratio of 50:1 for 30nm wide spectrum for LCs with birefringence ~0.3. If high birefringence (~0.4) LC becomes commercially available, then the reflectance and the contrast can be increased further to 75% and 60:1 respectively.
Chapter 6

Tunable LC/Polymer Composite Bragg Grating

6.1 Introduction

Photonic crystals\textsuperscript{90,91,92,93} which are characterized by periodically structured
dielectric material that can control the wavelength of reflected light have applications in
lasers\textsuperscript{94,95}, fibers\textsuperscript{96,97}, and telecommunications\textsuperscript{98}. Spatially periodic liquid crystalline
structures such as cholesteric liquid crystal (LC)\textsuperscript{2}, volume holographic polymer dispersed
liquid crystal (H-PDLC) grating\textsuperscript{99}, and colloidal crystals with LC cavities\textsuperscript{100} are examples
of naturally tunable photonic crystals. Highly tunable H-PDLC would have interesting
applications in reflective LC display devices and has a potential to be low cost high
efficiency alternative to current technology.

In this chapter a novel design for highly tunable liquid crystal/polymer composite
1D photonic crystal also known as Bragg grating is presented. Numerical models confirm
the tunability over the visible region ranging from 480nm to 620nm for mixed order
device. Modeled devices having active layers with $m\lambda/4$ and passive layers with $n\lambda/4$
thickness for the case of $m$ and $n$ odd integers are presented. The advantage of $m > n$ is
shown to be greater tunability for a given free spectral range. Possible applications of the
highly tunable device include but not limited to single pixel high efficiency reflective
color display device.
6.1.1 Bragg Gratings and HPDLC

Bragg grating based on H-PDLC in which alternating layers of polymer and LC rich regions result in high reflection has been considered for reflective display applications\textsuperscript{101,102}. However, H-PDLC has very low tunability and as a result stack containing at least three separate gratings is required for such application where broad wavelength tunability is necessary.

In a periodic modulation of high and low index dielectric materials with average index, $n_{ave}$, and period of the modulation, $P$ (See Figure 6.1), the interference condition for reflectance maximum is satisfied at the fundamental frequency corresponding to the wavelength at which the optical thicknesses of the dielectric layers are quarter-wave ($\lambda/4$) thick.

![Diagram of periodic modulation profile of refractive indices in 1D phase gratings](#)

**Figure 6.1**: Periodic modulation profile of refractive indices in 1D phase gratings
The optical stop bands of a quarter-wave stack and the spatial distribution of the refractive index profile are Fourier transform pair. Consequently discrete square-wave profile of refractive index can be represented by a Fourier series of which the first term has the same frequency corresponding to the fundamental stop band and the rest having the frequencies of the odd harmonics. Using Bragg condition of a reflection peak for normally incident light $\lambda = 2n_{\text{ave}}P^{24}$, tuning range of a simple quarter-wave stack of birefringent and isotropic dielectric layers can be calculated. Suppose layer one is passive thus $n_1$ is constant whereas layer two is active so that $n_2$ switches between $n_o$ and $n_e$, then in one state $n_{\text{ave}} = (n_1 + n_o)/2$, and in the other state $n_{\text{ave}} = (n_1 + n_e)/2$. In this case the tuning range is $\Delta \lambda / \lambda = \Delta n / (n_1 + n_o)$. Wavelength tunable 1D H-PDLC based on this approach with theoretical tuning range as high as 40nm in the visible spectrum was reported for LC material with birefringence $\sim 0.3^{103}$.

6.1.2 Reflective Single Pixel for Full Color Display Application

Current reflective color displays generate color with three pixels, where each one has a color filter photo patterned over it. In developing a high brightness reflective color display, an issue with this approach is that a maximum of 30% of the light can be reflected, and a dim display is inevitable. This issue can be solved by a device that consists of a stack of three cells each control a part of the spectrum. But the three cell solution is expensive and complicated.
A device where each pixel could be tuned to provide any color would solve the above problems. One concept is to have a pixel that can selectively reflect different colors as a function of an applied voltage, with some voltage where it is completely transmissive or absorptive with no reflection. This would allow for spectral colors. To obtain de-saturated colors, the pixels would need to switch quickly enough to be able to jump between about three colors in a frame time. This means the full color pixel would need to be able to change color as a function of voltage, and be able to switch in a few milliseconds.

Previously there have been ideas proposed for such voltage tunable color pixels. For instance, blue phase color generation proposed by Coles\textsuperscript{104,105}, variable cholesteric pitch proposed by Li\textsuperscript{106}, and tipping cholesteric helices by Lu\textsuperscript{107} are all examples of single pixel device that attempt to switch between multiple colors with an applied voltage.

The approach of Coles is related to the blue phase which has a defect structure whose periodicity causes the Bragg reflection of light. The periodicity of the defect structure can be altered by the application of an electric field. These devices have been demonstrated with a narrow bandwidth (about 25nm) reflection peak that provides high spectral purity. However, the tuning range that has been demonstrated is only about 80nm.

The approach of Li is to modify the pitch of a cholesteric using in plane electric fields. However, no data is given in his patent about the tunability of this approach. It
might be expected that it will be difficult to maintain a controlled bandwidth as a function of color tuning with this approach. Similarly, Lu's approach is to tip the cholesteric helices and create a blue shift.

At this point, it does not appear that there is a satisfactory method for producing the desired tunable color pixel that has fast switching. One of the problems with all of these approaches is that they generate their color through a changing of the periodicity of a structure, or by changing the orientation of a periodic structure. This approach is expected to have issues with large changes in either the period of the structure or its orientation. On the contrary, another way to approach the problem is to consider a device with a fixed periodic structure where the index of refraction of the structure is changed via nematic director reorientation. This approach could be expected to be more robust if it can be made and if the range of tuning is sufficient.

6.1.3 Increased Tunability LC/Polymer Composite Bragg Grating

In this chapter we propose a design for increased tunability Bragg grating that consists of alternating layers of LC and polymer material. Tunable reflective Bragg grating using LC have been proposed in the literature where the thicknesses of birefringent and isotropic medium were roughly the same. However, by changing the order of the LC layer in comparison to the polymer layer we show that it is possible to maximize the tunability for the peak reflection wavelength with the switching of LC index. To focus on the main design points for our device, we do not actually calculate the
LC director structure but rather model it as homogeneous active layers that have variable refractive indices in the LC/polymer Bragg grating simulation. Berreman’s 4x4 matrix method\textsuperscript{45,46} which is a direct solution of Maxwell’s equation in 1D was employed in calculating the reflection characteristics. In the calculation we ignored the effects of front reflection from the glass substrates and normalized the reflectance by the total incident polarized light.

6.2 Device Design

6.2.1 Principle of Operation

For a mixed order LC/polymer Bragg grating where the order and the wavelength at which the LC and the polymer layers are tuned differently, the wavelength at which reflection maximum occurs can be approximated as follows. Suppose the LC layers have an optical thickness of $m\lambda_{LC}/4$ and polymer layers have an optical thickness of $l\lambda_p/4$ where $m$ and $l$ are odd integers. Then the wavelength at which reflectance is maximum can be calculated as,

$$\lambda_0=(l\lambda_p+m\lambda_{LC})/(l+m)$$  \hspace{1cm} Eq. (6.1)

For example, Figure 6.2 shows reflectance spectrum of two cases where in one LC layers are $3\lambda/4$ at 480nm and polymer layers are $3\lambda/4$ at 550nm (blue curve), whereas in the other LC layers are $5\lambda/4$ at 480nm and polymer layers are $\lambda/4$ at 550nm (red curve).
Wavelengths at which reflectance is maximum for these two cases can be calculated using Eq. (6.1) to be ~510nm and ~490nm which agree with the simulation results.

Figure 6.2: Reflectance spectrum of LC/polymer phase grating \((n_{LC}n_p)^6n_{LC}\) where \(n_{LC}=1.8\), \(n_p=1.5\), \(n_s=1.52\)

We can divide the parameters that influence the tuning range of the device as static and dynamic variables. The static variables are the physical thicknesses of the polymer and the LC layers, index of the polymer, and the total number of layers in the stack. The dynamic variable by which the tunability is achieved is the LC index. We will consider the case where the polymer index is similar to the ordinary index of the liquid crystal. In this case we have the field on state, or the ordinary mode of the LC, corresponding to the low reflectance state; and the field off state, the extraordinary mode,
to the high reflectance state. As the LC index switches from ordinary to extraordinary mode, the optical thickness of the LC layers increases and the reflection peak tunes towards the longer wavelength region. At the same time reflection amplitude and bandwidth simultaneously increases as a result of increased mismatch in the indices of LC and polymer composite. These factors cause the reflection to be much higher in the red wavelength region and as a result it is preferable to tune the passive or the polymer layers to be reflective in the blue wavelength region in order to have a balanced reflection amplitude across the tunable spectrum.

6.2.2 Mixed Order LC/Polymer Composite Bragg Grating

To show the advantage of a mixed order device where \( l \neq m \), we will consider the case where \( n_p \approx n_o < n_e \). Then from Eq. (6.1), the minimum and maximum values of the reflected wavelength are given by

\[
\lambda_{\text{0 min}}^\text{max} = (l \lambda_p + m \lambda_{\text{LC-O}}) / (l + m) , \quad \text{and} \\
\lambda_{\text{0 max}}^\text{min} = (l \lambda_p + m \lambda_{\text{LC-E}}) / (l + m) .
\]

For clarity of argument, further assuming that polymer index is equal to the ordinary LC index, we can write the ratio of the maximum to minimum reflected wavelength as

\[
\lambda_{\text{0 max}} / \lambda_{\text{0 min}} = (l \lambda_p + m \lambda_{\text{LC-E}}) / (l \lambda_p + m \lambda_{\text{LC-O}}) = (l + m (n_e / n_o)) / (l + m) .
\]

From this relation we can see that if \( l=\text{m} \), then the range is independent of the order of the device. More importantly, we see that this ratio for the case of \( l=1; m=3 \), is about 1.5 times the case of \( l=1; m=1 \). Therefore a significant improvement in the tuning range is seen for the case of a mixed order device.
As a specific example, we can consider the case where the polymer index is $n_1=1.5$ and is constant while the LC index, $n_2$, switches between $n_o=1.6$ (ordinary state), and $n_e$ (extraordinary state); and find the required value of $n_e$ that will allow for a tuning range from 480 to 620nm. In the ordinary state we then want the reflectance peak to be at 480nm so that $(l\lambda_p+m\lambda_{LC-O})/(l+m)\sim480$ and we would like the extraordinary state reflectance peak to be tuned to 620nm so that $(l\lambda_p+m'\lambda_{LC-E})/(l+m')\sim620$. Here, $l$ is the quarter-wave order of the polymer layer, whereas $m$ and $m'$ are those of the ordinary and extraordinary LC states. The limited birefringence of the modeled LC keeps the order of ordinary and extraordinary states the same which means that $m=m'$. We will first assume that $l=1$, and consider the effect of the value of $m$ on the required change in the value of $n_e$ to achieve our desired tuning range. Then, in the ordinary state $(480+m\lambda_{LC-O})/(1+m)\sim480$ and in the extraordinary state $(480+m\lambda_{LC-E})/(1+m')\sim620$. From these two relations one can calculate the $\lambda_{LC-O}$ and $\lambda_{LC-E}$ for given LC quarter-wave order, $m$, and subsequently determine the LC layer thickness and the required extraordinary index. For example, if LC layer order is one so that $m=1$, then $\lambda_{LC-O}=480$ and $\lambda_{LC-E}=760$ and the LC layer thickness can be found from the relation $n_o d/480=m/4$ (where $n_o=1.6$, $m=1$) to be ~75nm. At this thickness the extraordinary index required to tune the reflection peak to 620nm is calculated using similar relation $n_e d/760=m/4$ (where $d=75$, $m=1$) and is found to be 2.53. As the LC layer thickness is increased to 3rd and 5th order ($m=3$ and $m=5$) the required extraordinary index to tune the reflection peak to 620nm is reduced to 2.23 and 2.16 respectively. We also modeled the case where the polymer layers are 3rd order ($l=3$) so that in the ordinary state $(1440+m\lambda_{LC-O})/(3+m)\sim480$ and in the extraordinary state...
In this case when $m=3$ the required extraordinary index is $\sim 2.53$ whereas when $m=5$ it drops to $\sim 2.35$. Note that we see here again, that in this approximation, the tuning range is independent of the order of the layers, if the orders of the layers are equal.

### 6.3 Numerical Model

#### 6.3.1 Low Polymer Order and Increasing LC Order

To verify these approximate results, we have performed a more detailed numerical calculation of some example Bragg gratings. Here we consider the device to be a stack of dielectric layers and calculate the reflectance spectrum using the Berreman 4x4 method which accurately takes into account the phase shifts at the layer boundaries. We have assumed that the dielectric indices are constant, and that the layers are attached to outer substrates with an index of 1.52. In the figures below the value of 1 is defined as total incident polarized light intensity.
Figure 6.3: Reflectance spectrum of LC/polymer phase grating \((n_{LC} n_P)^{27} n_{LC}\) where \(n_p = 1.5, n_s = 1.52, d_p \sim 75\text{nm}\); (6.3a: \(d_{LC} \sim 86\text{nm}\); 6.3b: \(d_{LC} \sim 234\text{nm}\); 6.3c: \(d_{LC} \sim 382\text{nm}\)
6.3.2 High Polymer Order and Increasing LC Order

In the previous section modeled Bragg gratings had 1st order polymer layers and liquid crystal layers increased from 1st to 3rd order quarter wave thickness. For comparison we would like to see the reflectance spectrum for higher order polymer layers and increasing order of liquid crystal layers.

Figure 6.4: Reflectance spectrum of LC/polymer phase grating \((n_{\text{LC}}n_{\text{P}})^2 n_{\text{LC}}\) where \(n_{\text{P}}=1.5, n_{\text{c}}=1.52, d_{\text{P}}\sim225\text{nm}\); (6.4a: \(d_{\text{LC}}\sim257\text{nm}\); 6.4b: \(d_{\text{LC}}\sim405\text{nm}\))

6.3.3 Alternative Designs for Display Applications

It is conceivable to have high index polymer layers in LC/polymer Bragg grating in which case slightly different design with better spectral reflectivities than before is possible. For example, Figure 6.5 shows the reflectance spectrum of LC/polymer Bragg
A grating where polymer index is 1.9 and LC index switches between 1.5 and 2.1. In this design the dark state is achieved by index matching at $n_{LC} \approx n_p \approx 1.9$. Unlike in the previous designs, reflectance cut-off near 550-600nm region resulted in spectral red ($n_{LC} \approx 2.1$) and green ($n_{LC} \approx 1.7$) reflectivities whereas strong index mismatch gives rise to increased blue ($n_{LC} \approx 1.5$) reflectivity.

Figure 6.5: Reflectance spectrum of LC/polymer phase grating $(n_{LC} n_p)^{28}$ where $n_p=1.9$, $n_s=1.52$, $d_p\approx 70nm$, $d_{LC}\approx 230nm$

**6.4 Discussion**

Figure 6.3 illustrates the reflectance spectrums of tunable mixed order device designs where the polymer layer order is fixed at 1 and the LC layer order is varied from 1 to 5. It can be seen that the best result for tunability is achieved when the order of the
LC layers is 5. In this case the required LC birefringence to tune the reflectance peak from 480nm to 620nm region is ~0.6. On the contrary in order to achieve the same tunability as the fifth order LC grating, first order LC grating requires birefringence ~1, and the third order LC grating requires birefringence ~0.7. However, it is important to note that when the order increases, the free spectral range shortens and the higher order peaks appear in the tuning region which limits the extent to which a higher value of $m$ can be used for a particular application. For example, in fifth order LC grating in Figure 6.3 higher order peak starts showing near 450nm region when the LC index is 2.1.

For comparison Figure 6.4 shows reflectance spectrums of phase gratings which have higher order passive layers than the gratings in Figure 6.3. As with the low order passive layers devices, increased order of LC layers require less LC birefringence for tunability. However, in this case free spectral range is reduced even further as the order of the whole assembly is increased and as a result device with fifth order active layers have higher order reflectance peaks in the tuning region.

From an applications perspective high tunability Bragg grating devices can be useful in reflective displays where single stack of LC/polymer layers replaces the individual red, green and blue pixel stacks of typical displays. In the previous examples the reflectance peak tunes between 480nm and 560nm using a liquid crystal with a birefringence of 0.3. A dark state could be achieved if the birefringence is increased so that matching between LC and polymer layers is possible. New nematic LC mixtures
with birefringence as high as 0.6 have been reported\textsuperscript{63} which would allow for a dark state and have a tuning range out to 620nm in our examples.

In the numerical simulations square-wave profile of refractive index modulation of LC and polymer layers gives rise to strong reflection side-lobes. In reality index mismatch at the interface of LC and polymer layers is not so sharp and as a result side-lobes are significantly suppressed. This effect of gradual index variation combined with the sinusoidal profile is used in dielectric mirror designs to get optical transmission stop bands without harmonics and with no side-lobes\textsuperscript{109}.

6.5 Summary

In conclusion, we numerically demonstrated the advantages of having mixed order LC/polymer phonic crystal in which the tunability is significantly increased when the LC quarter-wave order is higher than the polymer quarter-wave order. Previously LC/polymer Bragg grating technology where the order of the polymer and LC layers were the same exhibited switching of the reflection spectrum, but only a limited degree of tunability. We have shown that with a mixed order device, the tunable range of a Bragg grating can be significantly increased. Further, to make the free spectral range as large as possible, it is desirable for the order of the polymer layer to be one. A fabrication method for this device could be based on the H-PDLC approach, where coherent wavefronts interfere to provide a standing wave with periodicity along the cell normal. In the previously proposed devices, standing wave causes a periodicity in the formed polymer
rich layers, leaving alternating polymer rich and LC rich layers that are each about quarter of the wavelength of the coherent light source causing the standing wave. This fabrication method has been demonstrated by Bunning et al\textsuperscript{99} and subsequently Kato et al\textsuperscript{110} applied it for reflective display devices using 5\(\mu\)m thick grating with alternating layers of LC and polymer rich regions that has maximum reflectivity at \(\sim 16\text{V}/\mu\text{m}\) and no reflectivity at 0\(\text{V}/\mu\text{}\). However, in the case where the LC layer is \(3\lambda/4\) and the polymer layer is \(\lambda/4\) the wavelength of the standing wave would need to be doubled by using NIR illumination, and the ratio of polymer to LC be controlled so that the resulting polymer layer thickness is about 1/8 the wavelength of the NIR illumination. We expect that the reflectivity and the required voltage will be similar to that reported by Kato et al.
Chapter 7
Physics of Thin Liquid Crystal Cells

7.1 Introduction

7.1.1 Motivation for Thin Cells

Much of the earlier works concerning liquid crystals in confined geometries were focused on studying anchoring and switching properties in cylindrical and spherical geometries because of the growing interest in applications of PDLCs at the time. However, the observed effects of anchoring transitions and decrease of order parameter near the surface in such geometries can vary greatly from planar confinements, because curvature can have large effect on these parameters if the length scale of confinement is on the same order as the wavelength of the LC elastic distortion.

In the previous chapters we have concluded that in order for LC etalon-based projection display to achieve required efficiency with narrow bandwidth LED light sources, the cell thickness must be in sub-micron range. Vast majority of LC based optical devices along with research work that accompanies them cover cell thicknesses on the order of at least a micron where continuum models agree well with the experimental data. In addition to being useful in an etalon based devices, thin cavity LC would be able to achieve high speed switching compatible with other field-sequential display designs. It is known that the switching speed of an LC cell is proportional to the square of the
thickness of the cell. This means reduction of thickness by half would translate to a 4 times faster switching speed. However, this is true only within certain limits of thickness before the rise in the elastic distortion energy as well as the increasing importance of the surface anchoring conditions break the predictions of the continuum model.

Understanding of the limits of the approach of using thinner cells to provide faster switching, and the analysis of the limiting mechanism in terms of surface anchoring energy and surface order, are fundamental for new areas of applications. It is interesting to see the effects of weak anchoring in the anchoring transition and consequently the switching, or that of strong anchoring in the order parameter decrease near the surface. It is expected that with a weak anchoring condition the anchoring transition becomes the driving factor in switching, whereas in our case, where strong anchoring is present, decreasing order parameter might be the main factor.

In this chapter we explore order parameter measurements and electro-optic response in ultra thin LC cells. Previous researches on thin-cavity LC were focused on order parameter measurements, anchoring transitions, and director dynamics. First, we will briefly introduce the prior art on the topic of confined LC specifically focusing on decreasing order near the surface, anchoring transitions and director dynamics. These are parameters that are likely to have an important effect on electro-optic switching of thin cavity LC cells. After that we will present our work on studies of LC order and switching properties in sub-micron thin cells and will conclude with a summary of the current work and remarks on future work.
7.1.2 Previous Works

Among the works that have been published in the literature, general agreement is that LC ordering decreases near the surface and an anchoring transition is expected when the distortion energy of LC is much greater than the anchoring energy. For example, Uchida et al. calculated the order parameter as function of decreasing cell thickness by measuring the anisotropy of IR absorbance and found that the order parameter decreases near the surface of the cell\textsuperscript{111}. They measured the IR absorbance arising from the cyano group which aligns parallel to the director field. They observed that in the weak anchoring condition (PI alignment) the order parameter started decreasing from the bulk value of 0.6 at around 20nm from the surface. Similar experiment determining order parameter of thin LC films by measuring UV absorption was done by West et al\textsuperscript{72}. In their results, LC order parameter decreases sharply near 40nm to the surface and then rises up again gradually before falling back at around 20nm to the surface. They proposed that the sharp drop in order near 40nm was due to an anchoring transition from hybrid to homogeneous structure because of the increasing bulk distortion energy in comparison to the surface anchoring energy. In both Uchida and West's experiment decreasing order parameter near the surface qualitatively followed Landau-de Gennes theory as predicted by Yokoyama\textsuperscript{112}.

The confinement-induced anchoring transition in hybrid cells occurs at sufficiently short thicknesses leading to a uniform director configuration aligned along
the easy axis of the stronger anchoring. G. Barbero et al. calculated the anchoring strength and pretilt angle for hybrid aligned nematic cell by measuring the bulk birefringence as function of decreasing cell thickness\(^{113}\). Their calculation resulted in an extrapolation length of less than 0.4\(\mu\)m for strong anchored homeotropic surface, and that of around 1.3\(\mu\)m for a weakly anchored planar surface. Similarly, confinement induced anchoring transitions were reported and the critical thickness for the transition depends on the anchoring strength and varies from few \(\mu\)m for weak anchoring energies to about 100A for stronger anchoring energies\(^{114}\). It has also been reported that such configurational transition can be induced in homogeneous nematic LC in confined geometry that's under the influence of strong field where the field strength is, 
\[ E \sim (B/\varepsilon_0\Delta\varepsilon)^{1/2} \] (B is the coefficient of the cubic term in the Landau expansion) and the thickness of the LC is on the order of electric coherence length, 
\[ \zeta \sim (K/\varepsilon_0\Delta\varepsilon)^{1/2}/E^{115}. \]

7.2 Senarmont and UV Absorbance Measurements

7.2.1 Cell Fabrication

For the UV absorption measurements LC cells were constructed using a pair of quartz substrates on which an alignment layer, SiOx, was deposited at 30° angle for homogeneous alignment. Due to non-uniformity of the conducting ITO layers, an LC cell with thickness less than 1\(\mu\)m easily shorts making any electro-optic measurements a difficult task. As a result a protective layer of dielectric material is required for accurate measurements. First, we tried using liquid material to coat uniform film on the substrates.
We purchased a liquefied mixture from Nissan Chemicals (NHC-AT720A) that consists of SiO$_2$ and TiO$_2$ dissolved in an organic solvent. The mixture was spun down on substrates at speed of 2000rpm for film with thicknesses within the range of 500 to 1000A. After that substrates were baked at 300°C for 30 minutes in order to evaporate the liquid organic solvent. Although this method was relatively simple, layer thickness uniformity was not very good. Due to centrifugal forces the film in the center thinned gradually increasing towards the edge. In addition, the ultra-sonic cleaning processes that are required for alignment layer deposition damaged the film and some of the cells were shorted. As a result we switched to a dry process involving e-beam deposition of an SiO$_2$ layer on the substrates. UV ozone cleaning of substrates and ion beam bombardment prior to deposition results in relatively robust film capable of withstanding cleaning processes. Figure 7.1 shows the LC cell structure, where alignment layer and protective coatings have thickness around 50nm. Voltage drop across these layers is calculated in order to determine effective voltage of the LC layer.

Figure 7.1: LC Cell Configuration
We did not use any particle spacers for LC cell assembly. Instead a small amount of nematic LC, 5CB (Merck), was dropped on one substrate and the other substrate was put on it so that the capillary forces would pull the substrates together. Physical properties of the material are, $\Delta \varepsilon = 12$, $n_{\parallel} = 1.72$, $n_{\perp} = 1.53$, $\gamma = 0.03$PaS, and it has a nematic phase between $24^\circ$C and $35^\circ$C. Although cell assembly process was not perfected and thickness of the LC cavity was non-uniform, we were able to locate regions where thickness gradient was small enough not to interfere with the measurements.

7.2.2 Senarmont Measurements

The thickness of the LC cavity was determined using the Senarmont method and was complimented by UV absorption measurements in the case of ultra-thin cells. Experimental setup for the thickness measurement using Senarmont technique is shown in Figure 7.2. This method is extremely sensitive in measuring the sample birefringence from which thickness of the LC cavity can be determined. Although shearing or bending of a glass substrate produces a small birefringence that can be detected, it is usually insignificant compared to the large LC birefringence. However, for extremely thin LC cavities the measured birefringent is so small that any detection of birefringence from the substrates results in large error. In Senarmont method linearly polarized light goes through the birefringent sample and turns into elliptically polarized light which is converted by quarter-wave retarder back to linearly polarized light whose polarization is measured by finding the extinction angle with respect to the analyzer.
Sample birefringence is related to the extinction angle, $\phi$, of the analyzer as follows:

$$\Delta nd = \frac{\lambda \phi}{180} \quad \text{Eq. (7.1)}$$

In LC cells with ITO conducting layers it is possible to measure the cell thicknesses precisely by factoring out the contributions from bent glass using high field measurements. Figure 7.3 and Figure 7.4 show the plot of transmittance as a function of analyzer angle for cells filled with nematic LC, 5CB. When there is no external field applied across the cell, LC is homogeneously aligned and the measured extinction angle is approximately $1.5^\circ$ and $4^\circ$ respectively. However, when high field (10V) is applied across the cell, the extinction angle is not reduced to 0. In other words even when LC molecules are aligned homeotropically non-zero birefringence due to substrate and/or surface anisotropy is present. If the LC birefringence is calculated using the difference
between extinction angles in field on and off states, $\Delta \phi$, in Eq. (7.1), then the LC cavity thickness is around 10nm for cell in Figure 7.3 and around 50nm for cell in Figure 7.4.

![Transmittance Vs Phi](image1)

Figure 7.3: LC cell (c1227-1 from data122807)
Figure 7.5 shows the result of birefringence measurement for empty cell. Non-zero birefringence that was measured here is due to bending of glass substrates and/or anisotropic SiOx alignment layer that was evaporated on the substrates. Figure 7.6 shows birefringence measurement for cell filled with non-birefringent material (glycerol). Here, extinction angle is larger than the empty cell because in the process of filling, substrates got bent as capillary forces pulled them together in the region where glycerol is present.
Clearly when the measured extinction angle is around $1^\circ$, it is impossible to extract the LC birefringence from the experiment. In this range of thicknesses, measurement error can be very large as a result of non-zero birefringence of bent glass substrates and alignment layers. If the thickness is large enough such that birefringence caused by the substrates and the alignment layer is negligible, then Senarmont method can yield relatively accurate measurement. For example, if the measured cell thickness is around $0.5\mu m$ and the extinction angle is assumed to be accurate within $2^\circ$ to the correct value, then the measurement error is not more than $\pm 0.03\mu m$. As for very thin LC cells, UV absorption measurements can be used to determine the thicknesses.
7.2.3 UV Absorption Measurements

In addition to providing an alternate method for thickness measurement, UV absorption measurement can also help us to determine the LC order in extremely thin cells. Previously West et al. have attempted to measure the order parameter of LC near the surface by measuring the UV absorption of thin nematic films\(^1\). Although film thicknesses ranged from few nm to approximately 0.2\(\mu\)m, absorption measurements resulted in strange findings due to hybrid anchoring condition of homogeneous alignment layer and LC-air interface. In our case we carried out the measurements in extremely thin cells (few tens of nm at most) that have identical homogeneous anchoring on both surfaces.

LC cells were assembled using commercial grade fused quartz plates (GE Type 124) from Technical Glass Products, Inc of Cleveland. Transmittance of these plates is around 90\% from the visible down to about 270nm region (see Figure 7.7).
The plates were coated with ~50nm SiOx using high temperature evaporative coating chamber. During the coating process the chamber pressure was at $\sim 10^{-6}$ Torr, and SiOx deposition rate was $\sim 2.5\text{A/sec}$. $30^\circ$ obliquely deposited SiOx surface results in planar anchoring condition with no pretilt angle at the surfaces.

A Perkin Elmer spectrophotometer was used for absorbance measurements and the rough sketch of the experiment setup is shown in Figure 7.8. LC material is confined between UV transparent substrates and a metal mask lets light go through small uniform area where the absorbance is measured. In addition, UV polarizer was placed in front of the cell so that incident light is linearly polarized along or perpendicular to the LC molecular orientation depending on the LC cell position. LC absorbance along and perpendicular to the director axis is measured.
Absorbance of the material is proportional to the thickness of the material and is defined by Beer's law:

\[ A = \alpha lc \]  

Eq. (7.2)

Where, \( \alpha \) is the absorption coefficient, \( l \) is the material thickness, and \( c \) is the concentration of the material. Transmittance of light through the sample is related to the absorbance as:

\[ T = \exp(-A) \]  

Eq. (7.3)

UV absorption measurements can give us means to verify the thickness of the sample that was previously measured using Senarmont method. However, absorption coefficient, \( \alpha \), of the LC material is unknown. Although it is preferred to have information about anisotropic coefficients, \( \alpha_{\parallel} \), \( \alpha_{\perp} \), it is experimentally much simpler to determine the isotropic coefficient, \( \alpha_{iso} \). We measured the absorbance of dilute LC solution (~0.001%) of hexane using a 1cm quartz cuvette and calculated the isotropic coefficient using Eq. (7.2). Figure 7.9 shows the absorbance measurements. Solution exhibits strong absorbance at 275nm even when the LC concentration is very small. The isotropic extinction coefficient of 5CB was calculated to be \(~7.87\times10^{-3}\text{nm}^{-1}\). Once the isotropic absorption coefficient is known thickness of the LC cells can be calculated given that the isotropic absorbance is calculated from the average of anisotropic absorbances (see Eq. (7.4)).

\[ A_{iso} = \left(A_{\parallel} + 2A_{\perp}\right)/3 \]  

Eq. (7.4)
Note that for Eq. (7.4) to hold true, there must be no significant pretilt that would make the measured parallel absorbance less than the actual value. We use $30^\circ$ deposited SiO$_x$ alignment layer which results in homogenous anchoring with no pretilt.

![Absorbance Vs Wavelength](image)

Figure 7.9: UV absorption of dilute LC hexane solution

Absorbance of an empty cell was measured in order to make sure that there wouldn't be significant anisotropy in absorbance from the alignment layers (see Figure 7.10). Although a slight difference is seen in anisotropic absorbances of the empty cell, there is no significant difference when the absorbance is normalized by the subtracting base line absorbance measured at 350nm ($A_{\text{empty}}=A-A_{350}$).
Sample absorbance spectrums of ultra-thin LC cells are shown in Figure 7.11-Figure 7.13. We can calculate the LC absorbance at 280nm using Eq. (7.5).

$$A_{280} = \left[ (A_{280})_{LC} - (A_{350})_{LC} \right] - \left[ (A_{280})_{empty} - (A_{350})_{empty} \right]$$  Eq. 7.5

Once isotropic absorbances of ultra-thin cells are determined, the thickness of the LC layer can be calculated using Eq. (7.2) with concentration, $c$, being unity. Plot of the isotropic absorbance as a function of thickness is shown in Figure 7.14.
Figure 7.11: Absorbance of ultra-thin LC cell

Figure 7.12: Absorbance of ultra-thin LC cell
Figure 7.13: Absorbance of ultra-thin LC cell

Figure 7.14: Plot of isotropic absorbance as function of thickness for ultra-thin cells
Sample absorbances of relatively thicker LC cells are shown in Figure 7.15-Figure 7.16. For thick cells absorbance is notably larger than ultra-thin cells, and in the case of parallel absorbance it is completely saturated. As a result of absorbance saturation we're not able to calculate the thickness using absorbance measurements for cells with large thickness. Nevertheless we can plot the perpendicular absorbance of ultra-thin cells as function of thickness and extrapolate the data to calculate the thickness using the perpendicular absorbance of thicker cells. Figure 7.17 shows the plot of perpendicular absorbance of ultra-thin cells. Using the fit equation from Figure 7.17 we can calculate the thickness of LC layer given that the perpendicular absorbance is known. Table 7.1 shows comparisons between thicknesses from Senarmont and absorbance measurements. For thicker samples measurements have discrepancies as high as 18%. We believe such errors are result of inaccurate absorbance measurements due to saturation.

Figure 7.15: Absorbance of LC cell (d=0.51um)
Figure 7.16: Absorbance of LC cell (d=0.46um)

Figure 7.17: Plot of perpendicular absorbance as function of thickness for ultra-thin cells
Table 7.1: Comparisons between thicknesses calculated from Senarmont and absorbance measurements

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<th>(A⊥)experiment</th>
<th>d (Absorbance) μm</th>
<th>d (Senarmont) μm</th>
<th>% Difference</th>
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<td>0.46</td>
<td>0.51</td>
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<tr>
<td>1.8083</td>
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<td>1.55%</td>
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<td>0.39</td>
<td>0.46</td>
<td>17.8%</td>
</tr>
</tbody>
</table>

Absorption measurements can also give us information about the order of the LC in ultra-thin cells. Large anisotropy at around 280nm in perpendicular and parallel absorbances is a result of orientational ordering of the LC molecules. Therefore, dichroic ratio which is a representative of the order parameter is calculated using Eq. (7.6) where \( s' \) is approximately related to the order parameter \( s \) by some proportionality constant. By calculating \( s' \) we can say that the order parameter is more or less constant and equal to the bulk value down to about ~20nm and is reduced to less than half the bulk value at around ~15nm.

\[
R_{280} = \frac{(A_\parallel)_{280}}{(A_\perp)_{280}} = \frac{2s' + 1}{1 - s'}
\]

Eq. (7.6)

Figure 7.18 shows the plot of dichroic ratio as a function of LC layer thickness for ultra-thin cells.
Figure 7.18: Plot of dichroic ratio as a function of thickness for ultra-thin cells

7.2.4 Summary of Senarmont and UV Absorbance Measurements

The Senarmont method is extremely sensitive in measuring the sample birefringence from which thickness of the LC cavity can be determined. Although shearing or bending of glass substrate produces small birefringence that can be detected, it is usually insignificant compared to the large LC birefringence. However, for extremely thin LC cavities the measured birefringence is so small that any detection of birefringence from the substrates results in large error and as a result thicknesses in this region must be measured using UV absorption technique. In addition, for very thin cavities order parameter, thus the birefringence, $\Delta n$, of the LC is thought to be lower than the bulk value and therefore, an independent measurement for cavity thickness will be needed. Nevertheless, if the order parameter starts to change only near 20-30nm distance
from the surface, then our measurements should be reliable for cells thicker than these regions.

UV absorption measurements indicate that the LC order parameter is same as the bulk value down to almost ~20nm to the surface where it starts to decrease rapidly. We estimate that the order parameter drops to less than half of the bulk value within 15nm to the surface.

7.3 Liquid Crystal Switching

7.3.1 Background and Experimental Setup

According to the continuum model the switching time of a LC is proportional to the thickness of the cell and inversely proportional to the applied voltage.

\[
\tau_{\text{off}} = \frac{\gamma d^2}{\kappa^2 K}, \quad \tau_{\text{on}} = \frac{\tau_{\text{off}}}{\left(\frac{V}{V_c}\right)^2 - 1}
\]

Eq. (7.7)

A typical LC cell with thickness on the order of few microns has switching time of several msec. This means that according to the continuum model 0.01µm cell would have switching time of ~0.1µsec. However, we have to keep in mind that if the capacitative charge and discharge times of the LC cell are longer compared to the LC switching time, then we will not be able to measure the switching time accurately. RC time of a typical LC cell is estimated below for 0.01µm thick cell.
Although RC time is longer than the continuum model predicted switching time, we expect the actual switching time to be lot slower due to extreme elastic distortion energy of such thin cell which cannot be predicted by the continuum model.

Electro-optic response of LC cells was measured using the setup that is shown in Figure 7.19. Light transmittance through an ultra-thin LC cell placed in-between crossed polarizers is extremely low and requires very sensitive detector. In addition, such thin LC cells have much faster switching speed which necessitates faster detector response.

\[ RC = R \frac{AE_0 e_{LC}}{d} = 100\Omega \frac{10^{-4} m^2 \times 8.82 \times 10^{-12} F / m}{0.01 \times 10^{-6} m} = 10 \mu \text{sec} \quad \text{Eq. (7.8)} \]

![Figure 7.19: Experimental setup for the switching speed measurements](image)

In most gain bandwidth light detectors one usually has to sacrifice sensitivity in favor of response speed or vice versa. As a result 1/4\(\lambda\) plate was inserted in series with the LC cell
and polarizers. When 1/4\(\lambda\) plate is placed in series with LC cell, transmittance becomes 
\[ T \sim \sin^2(\pi/4 + \delta/2), \]
where \(\delta\) is retardation of LC cell. If \(\delta\) is small, then there will be greatest change in transmitted intensity with 1/4\(\lambda\) plate in series, because this brings the intensity halfway between minimum and maximum of sine function. If the light source is 35mW power laser, then crossed polarizers combined with 1/4\(\lambda\) plate would decrease the light intensity down to about 15mW (85% polarizer transmittance). Another 10% of the light will be lost going through the LC cell as a result of front and back reflection bringing down the total available power to 13mW. We can estimate the difference in transmitted intensity when LC is switched for the aforementioned setup as:

\[
T_1 = 13mW \times \sin^2\left(\frac{\pi}{4}\right), \quad T_2 = 13mW \times \sin^2\left(\frac{\pi}{4} + \frac{\pi \Delta n d}{\lambda}\right)
\]

\[
\Delta T = 13mW \times \left[\sin^2\left(\frac{\pi}{4}\right) - \sin^2\left(\frac{\pi}{4} + \pi \times 0.2 \times 10 / 632\right)\right] = 0.1mW
\]

This means that if the oscilloscope measurement accuracy is about \(\sim 1\text{mV}\), then the detector should have sensitivity of at least 10V/W.

Figure 7.20 shows the response speed of the detector measured by fast switching LED light. When voltage is applied across the LED light source, detector signal rises within \(\sim 2\mu\text{s}\) before saturating which means the detector speed is sufficient for measuring LC switching speed down to few \(\mu\text{s}\) range.
7.3.2 Switching Speed Measurements

At first we assembled LC cells with relatively large thickness using the method described in section 7.2.1. Specifically two cells with thickness around \(\sim 0.6 \mu m\) and \(\sim 0.4 \mu m\) were assembled for switching speed measurement. Figure 7.21-Figure 7.23 show the response speed of 0.6\(\mu m\) thick LC cell. Figure 7.24-Figure 7.26 show the response speed of 0.4\(\mu m\) thick LC cell. Turn-on and turn-off times were 200\(\mu s\), 800\(\mu s\) for 0.6\(\mu m\) cell whereas those were \(\sim 75 \mu s\), 300\(\mu s\) for 0.4\(\mu m\) cell.
Figure 7.21: Switching time measurement of thick LC cell with high field strength (d ∼ 0.6um)

Figure 7.22: Turn-on time ∼ 200µs (d ∼ 0.6um)
Figure 7.23: Turn-off time ~ 800µs (d ~ 0.6µm)

Figure 7.24: Switching time measurement of thick LC cell with high field strength (d ~ 0.4µm)
We see that for these two LC cells an increase in thickness by factor of 1.5 resulted in more than 2 times slower speeds. From these measurements we can conclude that
predictions of continuum model expressed by Eq. (7.7) are accurate when cell thickness is at least $\sim 0.4 \mu m$.

Next switching speed of a LC cell with thickness on the order of few tens of nm is measured (Figure 7.27-Figure 7.29). The thickness of the cell was estimated to be at most 0.1$\mu m$. Turn-on and turn-off times of this cell were estimated to be around 10$\mu s$ and 1.5ms respectively.

![Figure 7.27: Switching time measurement of ultra-thin LC cell with high field strength (d < 0.1$\mu m$)](image-url)
Figure 7.28: Turn-on time ~ 10µs (d < 0.1µm)

Figure 7.29: Turn-off time ~ 1.5ms (d < 0.1µm)
Figure 7.30-Figure 7.32 show the response of the same cell when applied field strength is lowered to 5Volts. Here, turn-on time is increased to ~40µs, whereas the turn-off time is decreased to ~80µs.

Figure 7.30: Switching time measurement of ultra-thin LC cell (d < 0.1µm)
In order to verify that the capacitance charge and discharge times were not affecting the thin cell switching speed measurements we measured the transmittance as function of voltage (TV) for different frequencies of applied field. The RMS voltage stays constant for the LC regardless of the applied frequency, whereas if the capacitance charge and discharge times are too slow in comparison to the field frequency then the LC will not switch. Figure 7.33 shows the TV curve for ultra-thin cell. Here, LC stops switching when period of the applied field is less than 100µs. Indeed, Figure 7.34 shows that the LC response no longer follows the applied field frequency and as a result average transmittance stays constant. In comparison, TV curve of thicker cell in Figure 7.35
shows that the LC is able to switch even when period of the applied field is decreased down to 25µs. Figure 7.36 where LC slightly switched confirms this result.

Figure 7.33: Plot of transmittance as function of voltage for ultra-thin cell (d < 0.1µm)
Figure 7.34: Response of ultra-thin cell when applied with high frequency field (d<0.1um)

Figure 7.35: Plot of transmittance as function of voltage for thick cell (d ~ 0.6um)

Figure 7.36: Response of thick cell when applied with high frequency field (d~0.6um)
We have also measured the transmittance as function of applied voltage for the ultra-thin cell to see if there is any hysteresis due to the anchoring transitions. As the cell gets thinner we expect there will be anchoring transition from planar to homeotropic alignment when applied voltage reaches certain threshold. If there is an anchoring transition effect, then it can be revealed in hysteresis in the transmittance as the voltage is ramped up and down. Figure 7.37 shows the TV curve for ramp up and down at ~85nm thick spot in the test cell. At first glance there isn't any hysteresis, however, when 1-3Volts region is magnified (see Figure 7.38) there seems to be slight discrepancy in transmittances for ramping up and down voltages. Repeated measurements yielded the same result which means that the slight hysteresis is not just a noise in the experiment.

Figure 7.37: TV curve (square wave w/ f=1000Hz)
In order to determine the cause of the lengthening of relaxation time we studied the LC texture during the switching process using polarizing optical microscopy. When we applied 3V to the test cell, hundreds of dark loops appeared (2\textsuperscript{nd} image in Figure 7.39) and they stayed even after the voltage was removed (3\textsuperscript{rd} image in Figure 7.39). After several seconds most of the dark disclination loops gradually disappeared through a slow dynamic process (3\textsuperscript{rd} image in Figure 7.39), although some were left permanently. The most possible cause of these disclination lines can be defect walls created by reverse tilting of LC molecules upon removing of the external field. If this is true then slight pretilt in the alignment would keep such walls from forming when voltage is removed from the LC cell.
Next we prepared the second test cell which was constructed in the same manner as the first one except this time alignment layer was rubbed by nylon material for slight anisotropy that would give rise to pretilt. We measured the thickness using Senarmont technique and found it to be around 126nm. This time there was no sign of any hysteresis in the TV curve (Figure 7.40) and the LC texture was without any disclination lines (see Figure 7.41).
Figure 7.40: TV curve of test cell 2 (d~126nm, transmittance was normalized to parallel polarizers and LC)
We measured switching speed for three different LC cells with thicknesses 0.6µm, 0.4µm and ~0.1µm. Table 7.1 summarizes the results of these measurements. Switching speed ratio between 0.6µm and 0.4µm cells is ~2.5 which is more or less consistent with the predictions of the continuum model (See Eq. (7.7)). Although turn-on time for ~0.1µm cell agrees with the model predictions, turn off time is unusually slow, especially for high field strength. LC texture analysis using optical polarizing microscopy
revealed that this is due to formation of defect lines as a result of reverse tilting of the LC director when applied field is removed.

Table 7.1: Switching speed measurement summary

<table>
<thead>
<tr>
<th>Thickness (µm)</th>
<th>T_on (µsec)</th>
<th>T_off (µsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6 µm (10 V)</td>
<td>200</td>
<td>800</td>
</tr>
<tr>
<td>0.4 µm (10 V)</td>
<td>75</td>
<td>300</td>
</tr>
<tr>
<td>~0.1 µm (10 V)</td>
<td>10</td>
<td>1.5ms</td>
</tr>
<tr>
<td>~0.1 µm (5 V)</td>
<td>40</td>
<td>80</td>
</tr>
</tbody>
</table>

7.4 Summary

In this chapter we studied LC order and electro-optic response in ultra-thin cells. UV absorption measurements were used to determine the thickness of ultra-thin cells. In order to verify the accuracy of the absorption measurement we checked it against the Senarmont measurement of thicker cells. However, for thicker cells UV absorbance was saturated and resulted in % difference as large as 18% in comparison with the Senarmont measurements. Dichroic ratio which is a representative of the LC order parameter was calculated for ultra-thin cells and clear downward trend of order parameter near the surface was found. It was consistent with previous findings where order parameter started to decrease at around 20nm to the surface.
LC switching speed for ultra-thin cell was measured and the turn on time was within the order of magnitude of the predictions of the continuum model. Slight discrepancies could be due to lengthened capacitance charge and discharge time. Reducing the resistivity of conducting layer would result in fast charge and discharge time. However, turn-off time was much longer especially in the case of applied high field strengths. This is due to formation of disclination lines that are caused by reverse tilting of the director upon removal of voltage.
Chapter 8

Conclusions and suggestions for future work

8.1 Summary of the Dissertation

In this dissertation I have shown numerical and experimental studies of LC etalon based optical modulators and tunable LC/Polymer composite Bragg gratings. In relation to the etalon device which requires submicron thickness LC cavity I have also conducted experimental studies on LC order and switching properties in ultra-thin cells.

- I have shown that the twist LC configuration within parallel mirror etalon device can be made polarization insensitive for select wavelength if the phases of the eigenmodes are tuned by varying the LC twist angle and cavity thickness.
- Polarization independent LC etalon can be used in transmissive optical modulator that can time sequentially tune select spectrum regions. The possible applications in pico-projection displays were discussed in the dissertation.
- Demo device for transmissive polarization independent LC etalon device was designed and built. Polarization independent tunability was confirmed by the device and the possible difficulties that might come up for projection display applications have been explored.
- Also it was shown that the polarization independent LC configuration can be used for reflective etalon modulator. Parts of the dissertation covered device design
that can be useful for a color pixel of a reflective display where in the one state select spectrum with 30nm bandwidth is reflected (~70%) and in the other state that spectrum region is transmitted.

- Design for tunable LC/polymer composite Bragg grating has been explored and the requirements for increased tunability from the perspectives of the relative LC and polymer layer orders have been presented.

- Finally, experimental studies of LC order in ultra-thin cells have been completed using UV absorption technique. Dichroic ratio which is a representative of the order parameter has been found to be decreasing rapidly starting at around 20nm to the surface which is consistent with the previous findings.

### 8.2 Conclusions

In conclusion, the thesis work has shown the basic requirements for polarization independent transmissive and reflective LC etalon devices and their applications as tunable optical modulators. More specifically, twist LC configuration within the cavity of parallel mirrors can be made polarization independent for select wavelength if the thickness and the twist angle are tuned right. The proposed transmissive LC etalon based modulator can be useful for narrow bandwidth LED or laser based pico-projection displays. On the contrary reflective LC etalon based modulator can be used as a color pixel of a reflective display.
It was also shown in the dissertation that the H-PDLC or the LC/polymer composite Bragg grating can be made highly tunable. The idea is that to have the tunable LC layer order higher order than the non-tunable polymer layer. However, the free spectral range reduces with the overall order of the device and as a result there is a limit of how high the LC order can go relative to the polymer order without compromising the tunability of the device.

8.3 Suggestions for Future Work

First of all for the transmissive LC etalon device to be useful the mirror characteristics need to be improved. The mirrors not only need to be absorption free but also the thin films stack design needs to be better tuned for a high contrast device. In addition, higher birefringence LC material and narrow bandwidth LEDs are required for the next demo device that can be seriously considered as projection display modulator.

As for the reflective device, current mirror stack design is optimized only within the working spectrum bandwidth of 30nm and as a result side-by-side pixel structure is a must in display applications. In order to be a serious contender as an efficient display device the mirrors need to be optimized such that out-of-bandwidth light is always transmitted. In this case stackable pixels can be made and the reflectivity would increase by three fold.
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